Math 601 Homework 8

Due Friday, October 26

Solutions should be typed or written neatly and legibly. Answers should be explained. You should reference all your sources, including your collaborators. For more information on writing up homework solutions, see the guidelines at the beginning of Homework 1.

Reading assignment:

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Sections 1.4–1.5, 1.7, 5.1–5.3

- From *Schaum’s Outline of Vector Analysis*:
  - Chapter 2

Required problems. Turn in a solution for each of the following problems.

1. Find the distance between the following two lines:

   \[
   \begin{bmatrix}
   x \\
   y \\
   z
   \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
   \]

   \[
   \begin{bmatrix}
   x \\
   y \\
   z
   \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
   \]

2. Consider the four points \(P_1 = (5, 2, 0), P_2 = (1, 5, -1), P_3 = (1, 7, 3),\) and \(P_4 = (4, 6, 3)\) in \(\mathbb{R}^3\).

   (a) Find the equation for the plane consisting of all points equidistant from \(P_1\) and \(P_2\).

   (b) Do the same for \(P_1\) and \(P_3\) and for \(P_1\) and \(P_4\).

   (c) Suppose that all four points lie on the surface of a sphere \(S\). Determine the center point of \(S\).
3. Consider the following system of partial differential equations:

\[
\frac{\partial u}{\partial x} = \frac{-xy^2}{(x^2 + y^2)^2} \quad \text{and} \quad \frac{\partial u}{\partial y} = \frac{x^2 y}{(x^2 + y^2)^2}
\]

(a) Given the above system, find equations for \( \frac{\partial u}{\partial r} \) and \( \frac{\partial u}{\partial \theta} \) in terms of the polar coordinates \( r \) and \( \theta \).

(b) Solve the system of equations that you obtained in part (a).

4. Consider the coordinates \( u, v \) on \( \mathbb{R}^2 \) defined by the equations:

\[
x = uv \quad \text{and} \quad y = \frac{1}{2}(u^2 - v^2)
\]

(a) Draw a sketch of the plane showing the gridlines \( u = -2, u = -1, u = 1, \) and \( u = 2, \) and \( v = -2, v = -1, v = 0, \) and \( v = 2 \).

(b) Determine \( \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial x}{\partial v}, \) and \( \frac{\partial y}{\partial v} \). Express your answers in terms of \( u \) and \( v \).

(c) Determine \( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \) and \( \frac{\partial v}{\partial y} \). Express your answers in terms of \( u, v, \) and \( y \).

5. Evaluate the integral \( \int \int_D 2y \ dA \), where \( D \) is the region bounded by \( y = x \) and \( y = x^2 \).

6. Compute the following integral:

\[
\int_1^e \int_{\ln x}^1 \frac{\sin(y^2)}{x} \ dy \ dx
\]

(Hint: Reverse the order of integration.)

**Recommended problems.** It is recommended that you do many more problems than the required problems. The following list of problems are good practice problems.

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Section 1.4: # 5–21 odd
  - Section 1.5: # 1–27 odd
  - Section 1.7: # 1–35 odd
  - Section 5.1: # 1, 3, 5, 9, 11, 13
  - Section 5.2: # 1–15 odd, 21, 23, 25
  - Section 5.3: # 1–13 odd

- From *Schaum’s Outline of Vector Analysis*:
  - Chapter 2: # 18, 19, 28, 29, 31, 32, 39, 45, 65, 67, 77–79, 82–84, 90, 95–97