Math 601 Homework 6

Due Friday, October 5

Solutions should be typed or written neatly and legibly. Answers should be explained. You should reference all your sources, including your collaborators. For more information on writing up homework solutions, see the guidelines at the beginning of Homework 1.

Reading assignment:

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Sections 6.1–6.3

- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Sections 11.1–11.5

Required problems. Turn in a solution for each of the following problems.

1. Find the eigenvalues for each of the following matrices. For each eigenvalue, find a basis of the corresponding eigenspace. Determine whether the matrix is diagonalizable over the complex numbers.

   (a) \[ A_1 = \begin{bmatrix} 4 & 6 & 4 \\ -2 & -3 & -4 \\ 0 & 0 & 2 \end{bmatrix} \]

   (b) \[ A_2 = \begin{bmatrix} 0 & -2 & 2 \\ 1 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix} \]

   (c) \[ A_3 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \]

   (d) \[ A_4 = \begin{bmatrix} 1 & -7 & -4 \\ -1 & 1 & -4 \\ 1 & 3 & 6 \end{bmatrix} \]

2. Solve the following system of differential equations:

   \[
   \begin{align*}
   x'' &= -x + 8y + 4x' - 2y' \\
   y'' &= -8x - 11y + 4x' + 8y'
   \end{align*}
   \]

   with initial conditions \( x(0) = 2, y(0) = -1, x'(0) = 4, \) and \( y'(0) = 7. \)
3. For a matrix $A$, the exponential of $A$ is the following matrix:

$$e^A = I + A + \frac{1}{2!} A^2 + \frac{1}{3!} A^3 + \frac{1}{4!} A^4 + \ldots$$

For each of the following matrices, compute the exponential of the matrix.

(a) $A_1 = \begin{bmatrix} -3 & 2 & -1 \\ -1 & 1 & 0 \\ 5 & -3 & 2 \end{bmatrix}$

(b) $A_2 = \begin{bmatrix} 0 & \phi \\ -\phi & 0 \end{bmatrix}$

(c) $A_3 = \begin{bmatrix} 5 & -2 \\ 3 & 0 \end{bmatrix}$ (Hint: Diagonalize the matrix.)

4. Two masses are connected by a series of springs between two fixed points as shown in the following figure:

Assume that the springs all have spring constant $k = 2.4 \text{ N/m}$ and that the masses have mass $m_1 = 0.3 \text{ kg}$ and $m_2 = 0.8 \text{ kg}$. Let $x_1$ and $x_2$ represent the displacements of the respective masses.

(a) Set up a system of second-order differential equations that describes the motion of this system.

(b) Solve the system given the initial conditions $x_1 = 6$, $x_2 = 2$ and $\frac{dx_1}{dt} = \frac{dx_2}{dt} = 0$ when $t = 0$.

Recommended problems. It is recommended that you do many more problems than the required problems. The following list of problems are good practice problems.

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Section 6.1: # 1
  - Section 6.2: # 1, 2, 4, 5, 6, 8, 10
  - Section 6.3: # 1–3 (Answers to these 3 problems will be posted to the webpage, as they are not in the back of the book.)

- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Chapter 11: # 1–4, 6, 9–22, 57–61