Math 601 Homework 5
Due Friday, September 28

Solutions should be typed or written neatly and legibly. Answers should be explained. You should reference all your sources, including your collaborators. For more information on writing up homework solutions, see the guidelines at the beginning of Homework 1.

Reading assignment:

• From Linear Algebra and Vector Calculus at Texas A&M:
  – Sections 3.5, 4.2–4.3
• From Schaum’s Outline of Beginning Linear Algebra:
  – Sections 6.1–6.4, 9.1–9.6

Required problems. Turn in a solution for each of the following problems.

1. Use Cramer’s Rule to solve for $x_1$, $x_2$, and $x_3$ in terms of $a$ and $b$.
   
   
   
   $$
   \begin{align*}
   ax_1 + x_2 - x_3 &= 0 \\
   5x_1 + bx_2 - x_3 &= 2 \\
   3x_1 - x_2 + x_3 &= 3
   \end{align*}
   $$

2. Consider the subspace $S$ of $C[0, 1]$ spanned by $\cosh x$ and $\sinh x$, and consider the following two bases of the subspace $S$:

   $$
   E = [\cosh x, \sinh x]
   $$

   $$
   F = [e^x, e^{-x}]
   $$

   (a) Find the transition matrix from $E$ to $F$.

   (b) Let $D$ be the differentiation operator on $S$; that is, $D (f(x)) = f'(x)$. Find the matrix representing $D$ with respect to the basis $E$.

   (c) Find the matrix representing $D$ with respect to the basis $F$. 
3. Consider the following linear transformation from $P_3$ to $P_2$:

$$L(p) = p'' + 2p'$$

Find the matrix representation of $L$ with respect to the bases $\{1 + x, 1 - x, 1 + x^2\}$ and $\{2 + x, x\}$.

4. Consider the following matrix:

$$A = \begin{pmatrix} -3 & 10 \\ -3 & 8 \end{pmatrix}$$

(a) Find a nonzero vector $u$ such that $Au = 2u$.
(b) Find a nonzero vector $v$ such that $Av = 3v$.
(c) Find the matrix representation for $A$ with respect to the basis $[u, v]$.

**Recommended problems.** It is recommended that you do many more problems than the required problems. The following list of problems are good practice problems.

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Section 4.2: # 12ad, 16ac
  - Section 4.3 # 1–6

- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Chapter 6: # 1–7, 12–16
  - Chapter 9: # 1–8, 10–13, 16, 18, 19