Math 601 Homework 4

Due Monday, September 24

Solutions should be typed or written neatly and legibly. Answers should be explained. You should reference all your sources, including your collaborators. For more information on writing up homework solutions, see the guidelines at the beginning of Homework 1.

Reading assignment:

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Sections 4.1–4.2, 2.1–2.3
- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Sections 8.1–8.9, 9.1–9.2, 10.1–10.3, 10.6–10.10, 10.14

Required problems. Turn in a solution for each of the following problems.

1. Let $P_4$ denote the vector space of all polynomials of degree less than or equal to 3. Consider the following linear transformation from $P_4$ to $P_4$:

   $$L(p) = p'' + \frac{1}{2}p' + p$$

   (a) Find the matrix representing $L$ with respect to the basis $\{1, x, x^2, x^3\}$.

   (b) Compute the inverse of the matrix in part (a).

   (c) Use your answer to part (b) to find one solution to the following differential equation: $y'' + \frac{1}{2}y' + y = x^3 - x$.

2. Let $S$ be the subspace of $C[0, 1]$ with basis $\{e^x, xe^x, x^2e^x\}$. Define a linear transformation $L : S \to \mathbb{R}$ by:

   $$L(f) = \int_0^1 f(x) \, dx$$

   (a) Find the $1 \times 3$ matrix representing $L$ with respect to the basis $\{e^x, xe^x, x^2e^x\}$.

   (b) Find a basis for the kernel of $L$. 

3. Use row reduction to compute the determinants of the following matrices:

(a) \( A = \begin{pmatrix} 1 & 1 & 3 & 2 \\ 1 & 3 & 11 & 12 \\ 2 & 1 & 3 & 6 \\ 4 & 2 & 5 & 8 \end{pmatrix} \)

(b) \( B = \begin{pmatrix} a & a & a & a & a & a \\ b & 2b & b & b & b & b \\ c & c & 3c & c & c & c \\ d & d & d & 4d & d & d \\ e & e & e & e & 5e & e \\ 1 & 1 & 1 & 1 & 1 & 6 \end{pmatrix} \)

4. For what values of \( a \) is the following matrix invertible:

\[
A = \begin{pmatrix} -1 & -1 & 0 & 4 \\ a & 0 & 4 & -3 \\ 4 & 2 & a & 1 \\ 1 & 0 & 1 & -1 \end{pmatrix}
\]

**Recommended problems.** It is recommended that you do many more problems than the required problems. The following list of problems are good practice problems.

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Section 4.1: # 18ac
  - Section 4.2: # 11, 13
  - Section 2.1: # 2, 3, 4, 6
  - Section 2.2: # 1, 3, 7
  - Section 2.3: # 1ac, 2, 3, 5

- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Chapter 8: # 75
  - Chapter 9: # 9, 33
  - Chapter 10: # 1–9, 13, 17–19, 21, 22, 24