Math 601 Homework 3

Due Friday, September 14

Solutions should be typed or written neatly and legibly. Answers should be explained. You should reference all your sources, including your collaborators. For more information on writing up homework solutions, see the guidelines at the beginning of Homework 1.

Reading assignment:

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Sections 3.1–3.5, 4.1–4.2
- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Sections 4.1–4.8, 5.1–5.7, 8.1–8.9

Required problems. Turn in a solution for each of the following problems.

1. Consider a $4 \times 6$ matrix $A$ with rank 4.
   (a) What is the dimension of the nullspace of $A$?
   (b) Are the columns of $A$ linearly independent?
   (c) Are the rows of $A$ linearly independent?
   (d) Do the columns of $A$ span $\mathbb{R}^4$?
   (e) Do the rows of $A$ span $\mathbb{R}^6$?
   (f) Suppose that $b$ is a vector in $\mathbb{R}^4$. How many solutions are there to the equation $Ax = b$?
2. Consider the following linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^3$:

- $L_1$ rotates each vector 90° about the $x$-axis (see the picture below).
- $L_2$ rotates each vector 45° about the $y$-axis (see the picture below).
- $L_3$ first rotates 90° about the $x$-axis and then rotates 45° about the $y$-axis.
- $L_4$ first rotates 45° about the $y$-axis, and then rotates 90° about the $x$-axis.

(a) Find the matrix representations of $L_1, L_2, L_3,$ and $L_4$.

(b) Compute $L_3 \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$.

3. Let $P_5$ denote the vector space of all polynomials with degree less than or equal to 4. Consider the subspace of $P_5$ consisting of polynomials $p(x)$ for which $p(1) = 0$ and $p(2) = 0$. Find a basis for this subspace.

4. Let $C[-\pi, \pi]$ denote the vector space of all real-valued functions that are defined and continuous on the closed interval $[-\pi, \pi]$.

(a) Consider the subspace of $C[-\pi, \pi]$ spanned by the vectors $\cos x$, $\sin x$, and $\sin(2x)$. What is the dimension of this subspace? Explain your answer.

(b) Consider the subspace of $C[-\pi, \pi]$ spanned by the vectors $\cos x$, $\cos(3x)$, and $\cos^3 x$. What is the dimension of this subspace? Explain your answer.
Recommended problems. It is recommended that you do many more problems than the required problems. The following list of problems are good practice problems.

- From *Linear Algebra and Vector Calculus at Texas A&M*:
  - Section 3.2: # 5, 14
  - Section 3.3: # 4, 6, 9
  - Section 3.4: # 11–15
  - Section 3.6: # 1, 2, 8
  - Section 4.1: # 5, 16ac
  - Section 4.2: # 2, 3, 4, 5

- From *Schaum’s Outline of Beginning Linear Algebra*:
  - Chapter 4: # 11, 12, 16, 21, 22, 43, 44, 53
  - Chapter 5: # 2, 5–8, 11, 18–20, 52, 72
  - Chapter 8: # 21–25, 29–32
  - Chapter 9: # 7, 30