Math 601 Example

1. Let $P_4$ denote the vector space of all polynomials with degree less 4. Consider the subspace of $P_4$ consisting of all polynomials $p(x)$ for which $p(2) = 0$. Find a basis for this subspace.

**Answer:** A polynomial in $P_4$ is of the form $p(x) = a + bx + cx^2 + dx^3$. If the polynomial is in the subspace, then $p(2) = 0$, which means:

$$a + 2b + 4c + 8d = 0$$

If we solve this equation for $a$, we get $a = -2b - 4c - 8d$. Thus, every polynomial in the subspace can be written in the form:

$$p(x) = (-2b - 4c - 8d) + bx + cx^2 + dx^3$$

We can rewrite this as:

$$p(x) = b(-2 + x) + c(-4 + x^2) + d(-8 + x^3)$$

Thus, every polynomial can be written as a linear combination of the polynomials $-2 + x$, $-4 + x^2$, and $-8 + x^3$. These polynomials are also linearly independent, so they form a basis.

Thus, a basis for the subspace is $\{-2 + x, -4 + x^2, -8 + x^3\}$. 