Math 308 Practice Test 5B

For the first three problems, do not use MATLAB to solve the differential equations or compute inverse Laplace transforms; in particular, you may not use the `dsolve` command or the `ilaplace` command (except to check your work). You may use MATLAB to compute integrals, solve equations, compute Laplace transforms, perform arithmetic, and check your answers. You must show your work to receive any credit.

1. Compute the following inverse Laplace transforms. (Show your work. Don’t use `ilaplace` except to check your answer.)

   (a) \[ \mathcal{L}^{-1}\left\{ \frac{s + 3}{s^2 + s + 1} \right\} \]

   (b) \[ \mathcal{L}^{-1}\left\{ \frac{-4}{s^3 - 2s^2} \right\} \]

   (c) \[ \mathcal{L}^{-1}\left\{ \frac{2e^{-2s}}{s^2 - 1} \right\} \]

2. Use Laplace transforms to solve the following initial value problem. (Your answer should be in terms of \( g(t) \).)

   \[ y'' + 5y' + 6y = g(t), \quad y(0) = 0, y'(0) = 0 \]

3. Solve the following system of differential equations:

   \[
   \begin{align*}
   x' &= x - y + t \\
   y' &= -4x + y
   \end{align*}
   \]

For the remaining two problems, you may use any MATLAB commands.

4. Consider the following system of differential equations:

   \[
   \begin{align*}
   x' &= (3x + 6)y \\
   y' &= (y - 2)(x - 1)
   \end{align*}
   \]

   Find all critical points (equilibrium points) for the system. For each equilibrium point, indicate whether it is stable or unstable.

5. A one loop RLC circuit consists of a 3-\( \Omega \) resistor, an inductor with \( L = 1.3 \) Henrys, a capacitor with \( C = 0.2 \) Farads, and an AC generator that produces a voltage of \( V = \sin(10t) \) volts. Set up a differential equation for the current \( I \).