Math 308 Practice Test 4A

For the first four problems, do not use MATLAB to solve the differential equations; in particular, you may not use the `dsolve` command (except to check your work). You may use MATLAB to compute integrals, solve equations, perform arithmetic, and check your answers. You must show your work to receive any credit.

1. Find all solutions to the following differential equation:

   \[ z'' + 2z' + 5z = 17 \sin t \cos t \]

2. Consider the following differential equation:

   \[ xy'' + (1 - 2x)y' + (x - 1)y = xe^x \]

   Two solutions to the corresponding homogeneous equation are \( y_1(x) = e^x \) and \( y_2(x) = e^x \ln x \). Find all solutions to the differential equation.

3. Use the Table of Laplace transforms to compute the Laplace transforms of the following functions.

   (a) \( f(t) = 2e^{-3t} \cos(2t) - 4t^4 + 5 \)
   
   (b) \( f(t) = 5t^4 + 4 \cos(3t) \)
   
   (c) \( f(t) = 3 \sin(2t) \cos(2t) + 6e^{5t} \)

4. Suppose that the Laplace transform of the function \( f(t) \) is

   \[ \mathcal{L}\{f(t)\} = \frac{s}{s^2 - 4}, \quad s > 2 \]

   Find \( \mathcal{L}\{e^{3t}f(t)\} \).

For the remaining two problems, you may use any MATLAB commands.

5. Consider the following initial value problem:

   \[ r' = \theta^2 - r^2, \quad r(-3) = 3 \]

   For each of the following, give your answer to 6 decimal places.

   (a) Use `eul.m` with stepsize \( h = .1 \) to approximate \( r(-2) \).
   
   (b) Use `rk2.m` with stepsize \( h = .1 \) to approximate \( r(-2) \).
   
   (c) Use `rk4.m` with stepsize \( h = .1 \) to approximate \( r(-2) \).
   
   (d) Use `dsolve` to find the actual value of \( r(-2) \) to 6 decimal places.

6. The Fourier cosine transform of a function \( f(t) \) is the function \( F(\omega) \) defined by the integral

   \[ F(\omega) = \int_0^\infty f(t) \cos(\omega t) \, dt \]

   Find the Fourier cosine transform of the function \( f(t) = e^{-t} \).