Math 308 Practice Test 3B

For the first four problems, do not use MATLAB to solve the differential equations; in particular, you may not use the `dsolve` command (except to check your work). You may use MATLAB to compute integrals, solve equations, perform arithmetic, and check your answers. You must show your work to receive any credit.

1. Solve the following initial value problem:
   \[ y'' - 2y' - 8y = 0, \quad y(0) = 1, \quad y'(0) = -8 \]

2. Solve the following differential equation:
   \[ x^2 y'' - 5xy' + 10y = 0 \]

3. Solve the following differential equation:
   \[ w''' - w'' - 5w' - 3w = 0 \]

4. Consider the following differential equation:
   \[ y'' + 3y' - 10y = f(x) \]
   where \( f(x) \) is some function of \( x \).
   
   (a) If \( f(x) = -8e^{3x} \), one solution is \( y_p = e^{3x} \). Find all solutions to the differential equation in this case.
   
   (b) If \( f(x) = 2 + 6x - 10x^2 \), one solution is \( y_p = x^2 \). Find all solutions to the differential equation in this case.
   
   (c) Find all solutions to the differential equation when \( f(x) = -8e^{3x} + 2 + 6x - 10x^2 \).

For the remaining two problems, you may use any MATLAB commands.

5. For each of the following pairs of functions, determine whether the functions \( y_1 \) and \( y_2 \) are linearly independent for \( x > 0 \).
   
   (a) \( y_1 = \ln(1/x) \)
   \[ y_2 = \ln(x^2) \]
   
   (b) \( y_1 = e^x \)
   \[ y_2 = xe^x \]
   
   (c) \( y_1 = \cos x \)
   \[ y_2 = 0 \]
6. Consider the following mass-spring system, in which a block lying on a surface (that has friction) is attached to a wall by a spring:

Let $x(t)$ be the distance of the block from its equilibrium position at time $t$. Suppose that $x(t)$ satisfies the following differential equation:

$$mx'' + bx' + kx = 0$$

where $m$, $b$, and $k$ are positive constants. Suppose the block is moved away from its equilibrium position and released.

(a) Describe the behavior of the block if $m = 1$, $b = 5$, and $k = 4$?

(b) Describe the behavior of the block if $m = 1$, $b = 1$, and $k = 4$?

(c) For what values of $b$ does the block oscillate (your answer should be in terms of $m$ and $k$)?