Math 308 Practice Test 3A

For the first five problems, do not use MATLAB to solve the differential equations; in particular, you may not use the `dsolve` command (except to check your work). You may use MATLAB to compute integrals, solve equations, perform arithmetic, and check your answers. You must show your work to receive any credit.

1. Solve the following initial value problem:
   \[ x^2y'' + 2xy' - 6y = 0, \quad y(1) = 5, \quad y'(1) = 0 \]

2. Solve the following differential equation:
   \[ z'' + 4z' + 5z = 0 \]

3. Solve the following differential equation:
   \[ y''' - 4y'' + y' + 6y = 0 \]

4. Consider the following differential equation:
   \[ xy'' + (1 - 2x)y' + (x - 1)y = 0, \quad x > 0 \]
   One solution to this differential equation is \( f(x) = e^x \). Find all solutions to the differential equation.

5. Consider the following differential equation:
   \[ y'' + 4y' + 5y = 20 \sin(5t) \]
   (a) Find all synchronous solutions of the form \( y = A \cos 5t + B \sin 5t \).
   (b) Find all solutions to the differential equation.

For the remaining two problems, you may use any MATLAB commands.

6. Compute the Wronskian of the functions \( y_1 = x \ln(\cos x) \) and \( y_2 = x \ln(\sin x) \).

7. Find a differential equation whose general solution is
   \[ y(x) = c_1 e^{2x} \cos(3x) + c_2 e^{2x} \sin(3x) \]