The Effect of Income Inequality and Residential Segregation on Infectious Diseases Transmission

A Senior Project submitted to
The Division of Science, Mathematics, and Computing
and
The Division of Social Studies
of

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Annandale-on-Hudson, New York
May, 2009

Abstract

Since the 1980s, income inequality and residential segregation have increased in industrialized and developing countries. There is a body of literature that hypothesizes and examines the relationship between inequality and segregation on health. However, none of these studies constructs a mathematical model to explain these complex linkages. This project builds a theoretical behavioral model that analyzes the effect of income inequality, neighborhood characteristics, and residential segregation on infectious diseases transmission. Residential segregation and income inequality decrease the community's health if an individual's health production function is concave with respect to income and exposure. We also discuss the possibility of a disease trap through the existence of multiple equilibria when disease externalities generate convexities in the health production function. Because a complete set of closed-form solutions to our model does not exist, we analyze our model qualitatively using techniques from Real Analysis.

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Acknowledgments

I would like to thank Professor Sanjaya DeSilva and Professor Sam Hsiao for their advice on my project, Professor Tsu-Yu Tsao for his suggestions, and Albert Kleine and Eliza Jarvinen for their help with my writing.

I also would like to thank my family and my friends for their support during my senior year at Bard.

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Introduction and Literature Review

The period from the 1980s has witnessed a concomitant increase in socioeconomic inequality and residential segregation. Household income inequality in the US increased during the 1980s and peaked in the early 1990s [14] [37]. England also experienced a rise in income inequality during this period [57]. Furthermore, income inequality is exhibited in a new dimension – residential segregation – that amplifies and reinforces its own existence. In the US, the proportion of poor people who lived in central cities was 34% in 1970, rose to 39% in 1980, and reached 43% in 1990. There was also a migration of the poor out of nonpoor neighborhoods. From 1970 to 1990, the percentage of central-city poor people living in nonpoor areas declined from 45% to 31%, while the percentage living in poor neighborhoods increased from 38% to 41%. Affluent concentration also became apparent. In 1970, a typical affluent person lived in a neighborhood that had 39% affluent population, but this figure rose to 50% in 1990. This geographic concentration and residential segregation result in increasing intraclass and decreasing interclass interactions [23].

In many cities in developing countries, gated communities have also seperated the rich and the poor physically and socially since the 1980s. For instance, a high rise tower called

Hamilton Court in Gurgagon, India protects its residents from surrounding slums with fences, security guards, a water system, a private school, and a private hospital [53]. In China, rural residents migrate to cities to seek employment, while urban citizens move to surburban gated communities and shield themselves from the ”uncivilized” population [26] [9]. In Brazil, the population living in close condominiums (CCs) doubled between 1998 and 2002 [32]. These enclaves locate mainly around the city suburbs and offer millions of middle to upper-class Brazilians high quality homes, a secured neighborhood, a clean environment, and a sophisticated leisure infrastructure. It is evident that income inequality and residential segregation have been a global trend. This geographic concentration and residential segregation inevitably create a new demographic and social outlook.

Humans live in society; thus any change in a community can potentially affect its peoples physical and social attainment. How do these community's characteristics - neighborhood income inequality, socioeconomic standards, and residential segregation - affect the quality of life? A review by Lynch et al. 2004 [22], Yen et al. 1999 [44], and Acevedo–Garcia 2000 [1] provide a quite complete picture of concurrent literature on the relationship between income inequality, neighborhood socioeconomic characteristics, and residential segregation on health. The rest of the chapter presents some of these relevant empirical studies and theoretical hypotheses.

1.1 Income inequality

The review paper by Lynch et al. 2004 includes cross-country analyses as well as examines the effect of income inequality on health in a country. All presented countries have high median income such as the US, the UK, Canada, Sweden, Taiwan, and Spain.

Cross-country Analysis

Cross-country analysis uses a country as an unit of study. Among the 26 international aggregate studies, 15 support the income inequality hypothesis, six find no association, and another five offer mixed support. Wilkinson 1992 [43] is an example of such support. He uses the Pearson correlation coefficient to examine the relationship between life expectancy (male and female at birth) and income distribution in nine countries in the Organisation for Economic Cooperation and Development. Income is sorted into tenth intervals, i.e. from the 10th percentile to the 100th percentile. The relative income coefficient is the ratio between each interval income and the accumulative income. For example, the relative income coefficient of the lowest 10th centile is 0.1 in a perfectly equal community. A coefficient that is less than 0.1 indicates that the lowest 10% of the population receives less than 10% of the total income, which implies inequality in the community. Controlling for gross national product per head (GNP per head), Wilkinson finds that there is a positive correlation between the relative income coefficient and life expectancy among tested countries. The relative coefficient has no effect on health in the 10th centile and the 20th centile and exhibits a negative and significant effect from the 30th to 100th centile. Because GNP per head is control for, he concludes that the association between income distribution and life expectancy is a result of factors associated with relative rather than absolute income [43].

There are also studies that do not find the importance of income inequality on health. Judge, Mulligan, and Benzeval (1998) [12] use the Luxembourg Income Study (LIS) dataset, which provides the internationally comparable data on income distribution data of 14 OCED countries, to examine the relationship between infant mortality and life expectancy on income inequality indicators. The indicators were calculated by Atkinson and colleagues, which is considered to be the most authorative and accurate measures. The specific indicators of income inequality that they employ are: 1. the share of income going to the bottom 10th, 20th, 60th, and 70th percentile of the income distribution (denoted S10, S20, S60, S70 respectively); 2. the share of income going to the top 5% of the income distribution (TOP 5); 3. the ratio of the 90th to the 10th percentile of the income distribution (P90/10); 4. the Gini coefficient. They demonstrate 3 different ways to examine the relationship between inequality and health: 1. partial correlation coefficents; 2. OLS to examine if the relation is still hold after controlling for other variables; 3. changes over time in income inequality and population health. Other control variables are national GDP, health expenditure, social security transfers, and the percentage of female in the labor force. A OLS regression of life expectancy on P90/100 shows no relationship between inequality and health before and after controlling for other determinants. In addition, there is no relationship between S60 and infant mortality after including control variables. Furthermore, none of the determinants shows any effect on health, except the impact of percentage of female in the labor force on infant mortality [12]. It is hard to compare this study with the previous one because they use a different data set and different income inequality indicators. Other studies that show no correlation between inequality and health mentioned in Lynch et al. 2004 are Lynch et al. (2001) [21] and Gravelle, Wildman, and Sutton (2002) [10].

Lobmayer and Wilkinson (2000) [19] conduct a study to explain why the studies after 1995 do not present an effect of income inequality on health. This study uses LIS, the same dataset as Judge et al. Their income data are from 1989-1992, while the data in Judge et al. 1998 were from various years in the 1980s. The number of deaths is reported by age groups in each country. The measure of income distribution is the ratio between the income of the 50th centile and the 10th centile. This index indicates the relative relationship between the bottom 10% and a country's mean income, i.e. 50th centile. A ratio that is higher than 1 indicates the existence of income inequality. It is because in a perfect equality case, the median income of the 50th centile and 10th centile are equal; therefore, the ratio is 1. Specfically, if the income at the 50th centile (the median income) was twice as high as that at the 10th centile (the top of the bottom centile), the 50:10 centile ratio would equal 2. In addition to income inequality, they also calculate national median income, age-specific absolute poverty, and age-specific relative poverty. Absolute poverty is defined by the proportion of people who are at the lowest bottom 10% of 14 countries. Relative poverty is defined by the proportion of people who are at the lowest bottom 10% of a country. National median income is calculated from household income based on the subjective equivalent scale and the OECD equivalent scale, which adjust for number of people in the house. For example according to the subjective equivalent scale , the first adult is counted as 1, the second one is 0.7, and any child from 0-1 year is counted as 0.2. They run a correlation coefficient between age-specific mortality and national median income, age-specific absolute poverty, age-specific relative poverty, and income inequality. They conclude that income inequality has a negative impact on young adult's health while shows no significant effect on the elderly (older than 65 years old). They claim that the reason why some cross–country analyses do not find the impact of income inequality on mortality is because they examine all age groups. However, this finding cannot explain results in Judge et al. 1998 because that study finds no impact of inequality on infant mortality after controlling for other determinants of health [19].

In general, it is hard to justify the results from cross-country studies because there are too many factors that can affect health outcomes. In addition, the dataset is rather small: it only includes 14 countries, which is well– below the minimum number of statistical significant observations, 30. Studies that report the negative impact of income inequality on health usually limit themselves to analyses of correlation coefficients and do not control for other variables. Therefore, they cannot answer questions of which factors of income inequality affect health and perhaps solve the causality problem: mortality leads to inequality, or inequality leads to mortality. As for the causality problem between inequality and health, it can be that because people are sick, they are poor. And the proportion of poor people increases income inequality. These studies do not control for other variables, they cannot claim whether or not and why income inequality leads to high mortality rates. Mortality rates can be determined by other factors such as the community's healthcare infrastructure and the level of education, which might be a result of or at least associated with income inequality. Among studies that shows no support for the impact of inequality on health, the association does not exist even before controlling for other determinants in some papers. There is also no common consensus on the dataset and the income inequality index should be used. As a result, it is hard to compare the two contradicting findings. In spite of their methodological limitations, cross-country analyses are the earliest studies that bring the issues of income inequality and health to our attention.

Within Country Cross-Section Studies

The US

Within country cross-section studies consider a community (city, state, and census tract) as an unit. The largest literature on income inequality are studies on the US. There are also inconsistent findings about the relationship between income inequality and an individual's health outcome. Lynch et al.2004 identify 24 within country cross–section study. Nine studies were supportive, four show no association, and other nine have mixed support. The two supporting studies by Kaplan et al. 1996 [13] and Kennedy et al. 1996 [15] are presented below.

A study by Kaplan et al. 1996 investigates the relationship between income inequality and health outcomes in 50 states in the US. They define income inequality as the proportion of aggregated household income held by households whose income is below a specified centile on the distribution of household income. Income inequality indicators are calculated for the 10th to the 90th centiles for all 50 states and for the US as the whole and find the correlation coefficient between mortality and these indicators. The correlation coefficient of a higher percentile is higher. Controlling for age, sex, and state median income, they find a strong correlation between all causes of death and income inequality. They conclude that a person who lives in a high income inequality area has a higher mortality rate than his counterpart in a less inequality community [13].

Kenedy et al. 1996 uses the Gini coefficient and the Robin Hood Index to measure income inequality. They examine the effect of income inequality on different causes of mortality among 50 states using Ordinary Least Square Method (OLS). They find that the Robin Hood index has a positive effect on mortality, including total mortality, infant mortality, heart disease, cancer, or infectious diseases. A community's healthcare expenditure, unemployment rate, crime rate, and education level are also associated with its income inequality. Results indicate that not only is the absolute level of income important, but the gap of income in the community is also crucial when discussing health outcomes. In addition, inequality might affect health via some community's factors [15].

As for studies showing no support, Lynch et al. 2004 summarize that these findings are after adjusting for some state characteristics such as education [25] or the proportion of African Americans residing in the state [7]. While the inclusion of education are absent in supporting studies, the proportion of black appears in some, such as Kennedy et al. 1996, and does not take away the independent impact of income inequality on. The studies showing mixed evidence tend to be conducted at lower levels of aggregation, such as counties, and indicate apparently conflicting findings.

Other countries:

Lynch et al. 2004 provide information on eight aggregate studies conducted in Canada and wealthy European countries and five in poorer nations. Stanistreet, Scott- Samuel, and Bellis 1999 [56] present the effects of income inequality on health in the UK . However, there is no evidence in time-series analyses for Canada as a whole [18] or within coastal communities in British Columbia [38]. Interestingly, Lorant et al. 2001 presents a correlation between higher income inequality and lower mortality in Belgium [20]. Inequality is not associated with long-term disability nor life expectancy after adjusting for average income in 17 regions in Spain [28]. Depending on the outcome and level of aggregation, the evidence is either negative or mixed in Brazil [36]. In Russia, the decline of life expectancy is linked to income inequality [40]. In Taiwan, a study by Chiang 1999 [6] shows that the association between income inequality and mortality of children under age was not significant in 1976 and 1985 but strengthened over time and became evident in 1995. However, this effect was weaker for total mortality.

Multi-level Studies

Multi-level studies take an individual as a unit of regression. Characteristics of a community in which the individual lives in are also taken into consideration. Among 25 multi-level studies in the US: nine are supportive, eight are negative, and other eight show a mixed result. The most consistently supportive evidence came from studies that measured income inequality at the state level [35]. When income inequality was defined at a lower level of aggregation, the evidence was less convincing [22]. Seven among the eight non-U.S. multilevel studies were conducted in high income countries. According to Lynch et al., Japan does not show any impact of income inequality on self-rated health [31]. This lack of association is also apparent on total mortality and ischemic heart disease (IHD) in Denmark, on self-rated health in Canada [24], and on mortality in New Zealand [5]. In the UK, mixed supports are indicated on self-rated health in Weich et al. 2002 [42] and on mental disorder in Weich et al. 2001 [41]. In contrast, Subramanian et al. 2003 finds that a greater income inequality associates with high odds of poor self- rated health in Chile

[34]. Their final reported results assess the effect of community income inequality (measured by the Gini coefficient) on self rated health, after controlling for household income, community median income and other key individual predictors such as age, sex, education level, and employment status. The effect of income inequality on health increases when the Gini is less than 0.45 and exhibits marginal diminishing returns when the Gini is greater than 0.45. Results suggest that community income inequality has an independent effect on health, over and above the well known effects of individual and household income.

In summary, there are inconsistent findings on the effect of income inequality on health. Cross-country analyses, within country cross-section analyses, and multi-level studies come to different conclusions. It is difficult to compare these studies because they do not always use the same measure of income inequality. Commenting on these studies methodology, a multi-level study appear to be the most appropriate method to examine the effect of income inequality on health because individuals are the ones who suffer from illness. On the other hand, studies on an aggregate level that compare places make a claim about an individual's outcome using community results, which can be methodologically justified but is not logically accurate. Many of the aggregate studies do not control for other neighborhood characteristics such as dilapidated housing and neighborhood educational level. These omissions obscure the relationship between income inequality and health because we do not know what factors of inequality affect health. The causality between mortality and income and inequality is not discussed. It might be that poor health causes low income, which leads to a high income inequality region. Therefore, they can only conclude that mortality and income inequality are correlated without further discussing a definite direction of this relationship nor why it is the case. Finally, choosing a right community unit is also important to examine to what extent income inequality has an effect on an individual's and community's health. In the US, the state unit shows a consistent effect in the multi-level studies after controlling for the state characteristics and individual characteristics. Taiwan's and Chile's studies report the effect at a city level.

1.2 Neighborhood Socioeconomic Status (SES)

According to Yen and Syme 1999, one group of epidemiologic studies has examined the health effects of community socioeconomic status, such as education, employment, and poverty. These studies have looked at the relation between living in an area with a certain set of socioeconomic characteristics and mortality risk, morbidity, and health behaviors. This body of literature also present inconsistent results. Similar to the studies on inequality, these studies can be categorized as within country cross-section analyses and multi-level studies. Regarding cross-section analyses, there are 17 studies that have reported an association between area socioeconomic status and all-cause mortality, four studies with cardiovascular mortality, and two with infant mortality without controlling for any individual SES. One criticism of these studies is that higher rates of disease in lower-SES areas might be caused by the effects of an individuals characteristics rather than those of the community [44]. In other words, low-SES areas would exhibit rates of poor health because people with low SES cluster in these areas.

Multi-level studies allow epidemiologists examine the effect of SES on health using a combined dataset that includes individual and community SES. In the US, community is usually defined at a census-tract level. A study by Hann et al. in the Alameda county is a example of multi-level studies [11]. They study the relationship between socioenvironmental risk factors with respect to mortality rates in Oakland, California. They compare mortality rates of all causes in poverty areas with those in non-poverty areas. A poverty area is characterized by the proportion of families with low income, the proportion of substandard housing, the contiguity of census tracts, the proportion of adults with low educational attainment, the proportion of unemployed, the proportion of unskilled male laborers, and the proportion of children in homes with a single parent. This poverty measurement partially include some neighborhood characteristics. Health data were collected by the Human Population Laboratory as part of a 19-year epidemiologic study in Alameda County, California, residents. Individual behavior such as smoking, educational level, and employment status act as control variables. Using a logistic model and control for all individuals characteristics, she finds that a poverty area has a significant effect on mortality rates. That is, a person living in a highly concentrated poverty area on average faces a higher risk of death than his counterpart in a low concentrated poverty area. In a broader sense, the environmental factors, not only individual characteristics, affect an individuals health. One drawback to this study is that it collapses all neighborhood characteristics in a measurement of poverty concentration; therefore, we cannot decompose which neighborhood characteristics contribute the most to an individual's health. In addition, her model only adds one variable at a time, which does not allow multiple factors to simultaneously explain the same health outcome.

Robert 1998 examines the relationship between the community level socioeconomic status (SES) and mortality rates, controlling for individual-level and family level SES on health [27]. Individual's and family's SES indicators are age, race, sex, education, income, and assets. Community-Level SES variables are age, race, sex, percentage of households receiving public assistance, percentage of families more 30,000 dollars of income (percentage of nonpoor household), and percentage of adult unemployment. Results indicate individual level and family SES indicators are strong predictors of health. Furthermore, a persons health is associated with SES characteristics of the community over and above one's own income, education, and assets, even though it is in a lesser extent than individual's and family's SES variables.

A unsupportive study is by Sloggest in England [33]. It is a longitudinal study that investigates the association between level of social deprivation in electoral wards and premature mortality among residents, before and after controlling for levels of personal deprivation. A random sample of nearly 300 000 people aged between 16 and 65 at the 1981 census and followed up for nearly nine years. Their health measurement is the number of deaths from all causes between ages of 16 and 70. Results indicate that without allowance for personal disadvantage, both sexes show a significantly positive relation between degree of residential deprivation in 1981 and premature death before 1990. For men, this association is effectively explained away after controlling for individual socioeconomic circumstances. For women living in wards of above average deprivation, the association is also effectively removed. They conclude that the excess mortality associated with deprived residential areas can be explained by the concentration in those areas of people with adverse personal or household socioeconomic factors.

1.3 Segregation

To my knowledge, there is only one study that examines the effect of income residential segregation on health as of now. A study by Waitzman and Smith in 1998 investigates the effect of income and residential segregation on mortality rates [39]. They also examine whether or not segregation has a linear impact on different income groups. The research was done for 30 metropolitan areas (MSA). The P index for isolation by Lieberson in 1980 is used as a measure of income inequality [58]. The P isolation measures the probability of intraclass interactions, i.e. the probability that a poor person will associate with another poor person and vice versa for the rich person. P index was calculated by the proportion of low income individuals and the total population in a census tract weighted by a proportion of low income individuals of that census tract and the total of low income individuals in the MSA. In other words, low income proportions in each subarea in an MSA are weighted by the proportion of all low income in the MSA. More specifically, we have

$$P_l = \frac{l_i}{T_i}\frac{l_i}{L}$$

Where P_l : isolation index of poor people

l_i : number of poor people in census tract i

T_i: total population in tract i

L: total poor population in the MSA.

Waitzman and Smith 1998 use both P indexes of concentration of poverty and affluence to indicate how segregated a community is. Notice that the P index of poverty and the P index of affluence do not need to be correlated. For example, a community can have a highly concentrated poverty population, but affluent people might or might not be concentrated. There are also cities that have both poor concentrated areas and rich concentrated areas, while other cities have only rich concentrated areas [54]. In this study, the poverty line is only defined in terms of income, which is different from the definition of poverty in Hanns study. Controlling for individual incomes and neighborhood incomes, the study indicates that high poverty concentrated areas exhibit higher mortality rates in comparison to low poverty concentrated areas. This is especially true for the young population. However, the P index of affluence does not have a robust effect on mortality throughout the regressions. Only when all control variables are included (individual and neighborhood incomes), the P index of affluence shows a negative and significant effect on mortality in comparison to an insignificant effect when incomes are not controlled for. This might indicate that neighborhood income and individual income have a positive effect on high-income people, which compensates the direct negative effect of segregation. In addition, the effect of residential segregation is not linear among all income groups. The poor shows the strongest effect with respect to both P indexes of poverty and affluence. The middle income group also shows a positive and significant effect of both poverty and affluence. The high income group does not show any significant effect.

The study also includes the MSA median income as their control variable. In this case, the neighborhood income has a negative effect on the mortality rate. This implies that a person living in a poor neighborhood would have a higher mortality rate than if he was living in a rich neighborhood. Thus, these findings are consistent with the finding in Hanns study. To summarize, a person who lives in a poor neighborhood has a higher mortality rate, and a person living in a segregated community (either poverty concentrated and affluence concentrated) also has a higher mortality rate. This effect is significant among poor and middle-income families. One limitation of this study is that it does not include other neighborhood characteristics such as dilapidated housing, education level, and access to healthcare facility. Therefore, one cannot decompose the factors among these effects. In addition, they do not control for race, which appears to be an important factor when studying analysis income inequality and segregation in the US.

Other studies on segregation deal with issues of racial segregation. The majority of studies examine the effect of racial residential segregation on the health outcomes of African Americans [1]. The general finding is that black mortality is positively associated with residential segregation and with residence in predominantly black areas. One example of such studies is by Cooper et al in 2001 [45]. Residential segregation is measured using an index of dissimilarity, representing the unevenness of the black-white racial distribution of households by census tract. A single measure of residential segregation is generated for each MSA. This indicator also has a range from 0 to 1, with a maximum value reflecting complete segregation.

This paragraph concludes the empirical review section on the effect of income inequality, community's socioeconomic status, and residential segregation on health. The findings are inconsistent. These studies do not take into account of causality between income and health; as a result, they might omit a potential causality between inequality, segregation, and health. In addition, most of the above studies have been done in rich nations where infectious diseases are not prevalent. Since chronic diseases are not spread from persons to persons, inequality, neighborhood characteristics, and segregation seem to have less of an effect on an individual's health comparing to that of infectious diseases. Yet, many studies show a relationship between inequality, community's characteristics, segregation, and health. This gives us a reason to suspect that in a poorer country where infectious diseases are prevalent, these factors can play an important on the country's public health issues.

1.4 Possible pathway that inequality and segregation could lead to high disease incidence

The income inequality-health hypothesis has been described in various ways. According to Lynch et al. [22], Wagstaff and van Doorslaer (2000) [46] define different versions of the income inequality-health hypothesis: the absolute income hypothesis (AIH); relative income hypothesis (RIH); deprivation hypothesis (DH), which is a variant of the RIH; relative position hypothesis (RPH); and income inequality hypothesis (IIH). The AIH predicts that there is no association between income inequality and health after controlling for absolute income at the individual level. The RIH predicts that it is income relative to some social group average that is important. The DH predicts that it is income relative to poverty that is important to health. The RPH proposes that it is an individual's position in the income distribution that matters. The IIH suggests that there is a direct effect of income inequality on health after control for absolute income. Mellor and Milyo (2002) further described strong and weak variants of the IIH in regard to its predictions about income transfers [47]. The strong version argues that for two individuals, A (high income) and B (low income), a transfer of income from A to B will improve the health of both.

The weak version of the IIH suggests that such an income transfer will improve the health of B much more than will the reduction of health for A. As a result, the transfer benefits the society as the whole.

In addition, Lynch and colleagues (2000a) offered three interpretations of evidence linking income inequality and health: the individual income, the psychosocial, and the neo-material interpretations [48]. The individual income is the same as AIH described above. As for psychological effect, there are also a strong and a weak version. A strong version states that direct health effects of income inequality represent general psychological processes that are among the major determinants of population health in rich countries. The weak version states that the direct health effects of income inequality represent particular psychosocial processes that influence some health outcomes in rich countries. The neo-material hypothesis states that the direct health effects of income inequality result from the difference of accumulation of exposures that do not result directly from perceptions of disadvantage. In addition, income inequality might lead to a lower social cohesion community. Lower levels of community attachment may result in greater reluctance to invest in such health-promoting human capital, such as education and medical care [13]. Income inequality may result in a government's underinvestment in human resources [49].

Dolores Acevedo-Garcia in "Residential segregation and the epidemiology of infectious diseases" explains possible direct and indirect pathways that residential segregation results in high mortality [1]. Indirect pathways refer to the effect of segregation on the quality of the living environment inhabited by the segregated group, i.e. segregation may result in poverty concentration, overcrowding, housing dilapidation, social disorganization, and limited access to healthcare. For instance, many studies have confirmed a positive association between low socioeconomic status and tuberculosis (TB) in contemporary populations. Such studies are Cantwell et al. 1994 [50] and Spence et al. 1993 [51]. Housing conditions are also important. TB is spread through coughing or spitting by individuals who suffer from active TB. Therefore, the degree of physical contact between infected and non-infected individuals, and consequently the extent of overcrowding and lack of ventilation at home and in workplaces are key factors in TB transmission. Another indirect pathway is social disorganization. This leads to some risky behaviors such as smoking and drug usage. For instance, crowded housing and coughing associated with smoking increase the chance of contracting TB.

The direct pathways can be expressed in three possibilities: spatial distribution, contact patterns between the segregated group and the rest of the population, and density of susceptibles. Since most of the infectious diseases infections require close contact between infectious and susceptible individuals, the spatial distribution of individuals and their movements across spatial locations determine disease spread and persistence. Berkman and Glass 2000 analyze the network analysis which acknowledges that the spread of infectious disease does not occur randomly but through socially constructed networks. By limiting who interacts with whom, residential segregation may shape the structure of social networks, and, consequently, may influence the transmission and distribution of infectious diseases across various groups in society. Density of susceptible individuals also affect the transmission. Higher densities accelerate transmission because they increase the rates of contact between infectious and susceptible individuals.

It is important to separate between the direct and indirect effect and examine the magnitude of each effect on population health. If a direct effect of inequality and segregation on health is large, the government has to make an effort to redistribute wealth. On the other hand, a high income inequality and segregated neighborhood might experience an underinvestment in public goods and services, such as a healthcare facility and a sanitary system. If this poor living environment has a major impact on the community's members, the government's priority is not to close the income gap itself but rather to improve the neighborhood quality. A concrete example is the case where poor and sick people are concentrated in an area, while the high income group lives in another community. If this segregation manifests the disease prevalence simply because the vulnerable population (the low-income group) lives in a high exposure environment, then the government should alleviate the disease burden by decreasing the density and concentration of poor people in that area. However, if disease transmission increases because people in the poor area do not get access to clean water, then the first focus should be to improve the community's facility, rather than migrating people from one area to another. The direct effect and indirect effect lead to different policy implications. As a result, it is crucial to separate them.

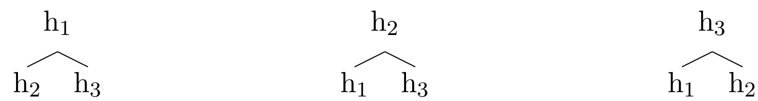
The rest of the project builds a theoretical behavioral framework that analyzes how income inequality and residential segregation can directly affect infectious disease transmission. The model does not include other socioeconomic factors associated with inequality and segregation that might influent dynamics of diseases, such as access to healthcare facilities or a good sanitary system.

2
General Framework

2.1 Model's Justifications

Following are some justifications for the model.

Scenario 1: Infectious diseases are usually characterized as airborne (tuberculosis and influenza), vector-borne (malaria), waterborne (cholera), and sexual transmitted disease (HIV/AIDS and syphilis). Many diseases are transmitted via the interaction between persons. If person i lives near person j, then person i's health (denoted h_i) is affected by person j's health (denoted h_j where $i \neq j$), and vice versa. One person's health depends on the health level of people around him. The health of people who he is not associated with does not affect his well-being and vice versa. We can define the person i's environmental health due to the health level of people he is associated with and the degree to which he is connected with person j as H_i and k_{ij} respectively. Therefore, person i's environment health can be thought of as a sum of all these connections, i.e. $H_i = \sum_{j=1,j\neq i}^{n} k_{ij}h_j$. As a result, the health level of a community is individual specific. In other words, the surrounding health of each person in the community is different, or $H_i \neq H_j$. To demonstrate this point further, let us examine a community of three people where their health levels are h_1, h_2, and h_3.



The environmental health of person 1 is $H_1 = k_{12}h_2 + k_{13}h_3$. Similarly, the environmental health of person 2 is $H_2 = k_{21}h_1 + k_{23}h_3$. It is clear from here that $H_1 \neq H_2$ because person 2 and person 3 might have a different contact than that of person 1 and person 3 ($k_{13} \neq k_{23}$). Therefore, the environmental health of each person is individual specific. Ideally, we want to capture all of these complicated and interconnected relationships, but it is hard to model when everyone possesses a different environmental health. Scenario 2 presents a more simple way to look at the problem.

Scenario 2: For simplicity, we can assmume that each person shares a common environmental health stock H. In this case H resembles a public good where one person cannot prevent others from contributing to this common health stock. The more participants there are, the larger this common good is. Specifically, if many people in the community are healthy, then the community's health stock increases. This increase in health stock in turn creates a positive externality to its people. The common health stock is now shared among people and becomes a common property of everyone in the community. The surrounding environment is no longer individual specific. Rather, it is community specific: as long as you live in the community, you share this health stock with everyone else. The community's sanitary system is an example for such common health stock. If the system is good, everyone has clean water. And if the system is bad, everyone has to use dirty water. In this scenario, there are two subscenarios:

Subscenario 1: The community can exhibit two characteristics

1. the more you are exposed to the common health stock, the more you contribute to it.

2. The more you are exposed to the common health stock, the more you are affected by it.

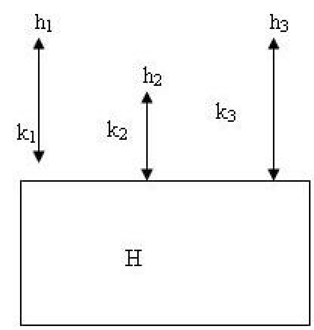


Figure 2.1.1. A two-way relationship between an individual's health and the health of his community via his exposure coefficient

This scenario is analgous to some local environmental pollution issues. For example, imagine there is a lake in your area. The distance between your house and the lake determines how much you pollute the lake and how much you are affected by such pollution. If you live close to the lake, you are likely to pollute it (dumping garbage to the lake). At the same time, you are more likely to get affected by your pollution and other people's pollution (bad smell and bacteria). In other words, there is a two- way relationship between the common health stock and an individual. The closer the person is, the more he contributes to the stock and the more he is influenced by it.

Subscenario 2:

This scenario decribes a one-way relationship between the health stock of the community and an individual's health. The closer you are to the health stock (high k_i), the more likely

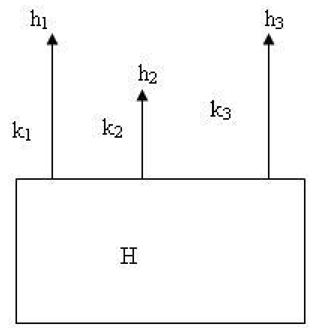


Figure 2.1.2. An one-way relationship between an individual's health and the health of his community via his exposure coefficient

you get affected by this pool. However, the fact that you live closer to the common pool does not mean that you directly contribute more to the common pool. Let us examine a case of a waterborne disease, cholera, in a village. Everyone in the town shares the same source of drinking water and a sanitary system, which are a common pool for waterborne diseases. However, some people in the town have a water filter that can filter out bacteria from drinking water. As a result, these people have a low exposure to diseases. But since everyone's sewage goes to the same place, everyone's direct contribution to the disease pool is the same regardless how high or low exposure the disease is. Notice that people with low exposure to contaminated water, because they have a water filter, indirectly contribute less to the disease pool because they are healthier. This is different from subscenario 1 where these people are less exposed to the disease pool and at the same time directly contribute less to the disease pool. In other words, k_i only reflects how likely person i gets affected by the common pool, but it does not represent how likely this person contributes to the pool.

For simplicity, we choose subsenario 2 to build our model.

In this model, we consider an individual as a utility maximizer. The utility maximization problem is the basic framework of this model. A utility function is a way to assign a value to all possible consumption bundles such that more-prefered bundles get assigned larger numbers than less-prefered bundles. A utility is a way to express preferences. Ordinary and cardinal utility are two types of utility. Ordinary utility emphasizes on how the bundles of goods are ordered. The magnitude of the difference in utility between any two consumption bundles is not important. On the other hand, cardinal utility investigates the size of the utility difference between two bundles of goods. For instance, in ordinary utility, one only asks whether or not one person prefers a bundle to the other. Cardinal utility asks how much first bundle is preferred to the second one. In economic analyses, we only consider ordinary utility, that is, the order in which bundles of goods is ranked. Economists informally refer to utility as a degree of happinnes: the higher one's utility is, the happier he is [61]. The utility maximization method is common in economics modeling. Given one's constraint, the goal is to maximize one's utility. The method solves the allocation problem among inputs in the utility function in order to maximize one's utility. At first, it might be hard to imagine how in real life one can maximize his utility, because one cannot measure or express his utility in a concrete number and unit. However in everyday life, people always make a tradeoff between one activity and another, choose the most preferred bundle, and make decisions that are the most beneficial to them. We can view the utility maximization method as such activities.

In this model, one person's utility U_i is a function of his health h_i and other consumption goods c_i. A person is happy when he is healthy and has goods to consume. Consumption goods are defined in a continuous term c_i. So c_i is not a discrete state of either having a TV or not, rather c_i can be thought as a continuous variable as it represents an aggregate of all consumption. For instance, one does not necessary know what $c_i = 10$ units of consumption or $c_i = 9.5$ units of consumption represents. But one knows that 10 units of consumption is higher than 9.5. Therefore, c_i takes a value in $[0, \infty)$. Analogously, one's health level h_i is a continuous variable in $[0, \infty)$, and its unit is an unit of health.

In short, we want to maximize an individual's utility where utility consists of health level and other consumption goods. Health is itself a production function whose inputs are medicine x_i, the community's health H, and the individual's exposure to the surrounding environment k_i. We can think of k_i as an extra quantity of medicine one would gain by living near the health stock H. A person who has a high exposure to the health stock (high k_i) would be healthier than his counterpart who possesses a low exposure (low k_i). Exposure k_i has unit of medicine per unit of health and takes values between 0 and 1. $k = 0.4$ means that 1 unit of community health H is equivalent to 0.4 unit of medicine, and $k_i = 0$ indicates that the health of person i is independent from the community's health H. At the same time, this person faces a budget constraint since he only earns a certain income per year and needs to allocate this money between buying medicine to improve his health and purcharging other consumption goods. Our goal is to examine the relationship between the indiviual's health and the community's health stock, income inequality, and residential segregation. We summarize our variables:

Utility function: $U_i = U_i(c_i, h_i)$ where

U_i: the utility function of person i

c_i: consumption goods of person i, $c_i \in [0, \infty)$

h_i: health production function of person i, $h_i \in [0, \infty)$

Health production function: $h_i(k_i, x_i, H)$ where

k_i: the degree in which person i is exposed to the environment, $k_i \in [0, 1]$

x_i: medicine that person i takes, $x_i \in [0, \infty)$

H: an average of the community's health stock. $H = \frac{\sum_{i=1}^{n} h_i}{n}$

Budget constraint: $c_i + px_i = I_i$ where

p: normal price of medicine with respect to consumption goods c_i

I_i: income of person i.

Then we are interested in maximizing $U_i[c, h(k_i, x_i, H)]$ with respect to c_i and x_i subject to the constraint $c_i + px_i = I_i$

Solving for the maximization problem, we show that under a certain general assumptions, we have a solution x_i^*, c_i^*, h_i^*, and H^*, which can be expressed in terms of k_i, I_i, p, H

The ultimate goals are the following:

(1) Compare an individual's health h_i in an income inequality community versus an equality community.

(2) Examine an individual's health production function with respect to individual's income and neighborhood income, i.e., $\frac{\partial h_i^*}{\partial I_i}$ and $\frac{\partial^2 h_i^*}{\partial I_i^2}$ in an intergrated community and segregated community.

2.2 The Model: General Form

In this subsection, we exclude all indexes. All notations (U, c, h, etc) are at an individual level unless otherwise stated. First, let us recall some theorems in Real Analysis that we will use throughout this section

2.2.1 Mathematical Background

Theorem 2.2.1 (Intermediate Value Theorem). *Let $[a, b] \subset \mathbb{R}$ be a closed bounded interval, and let $f : [a, b] \to \mathbb{R}$ be a function. Suppose that f is continuous. If $r \in \mathbb{R}$ is strictly between $f(a)$ and $f(b)$, then there is some $c \in (a, b)$ such that $f(c) = r$*

The Intermediate Value Theorem in this section is the same as Theorem 3.5.2 in Real Analysis by Bloch [59]

Theorem 2.2.2 (Mean Value Theorem). *Let $[a, b] \subset \mathbb{R}$ be a non-degenerate closed bounded interval, and let $f : [a, b] \to \mathbb{R}$ be a function. Suppose that f is continuous on $[a, b]$ and differentiable on (a, b). There is some $c \in (a, b)$ such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Theorem 2.2.2 is the same as Theorem 4.4.4 in Real Analysis book by Bloch [59]

Theorem 2.2.3. *Let $I \to \mathbb{R}$ be an open interval, let $c \in I$, and let $f : I \to \mathbb{R}$ be a function. Then f is continuous at c if and only if $\lim_{x \to c} f(x)$ exists and $\lim_{x \to c} f(x) = f(c)$*

Theorem **??** is the same as Theorem 3.3.2 by Bloch [59]

Theorem 2.2.4 (Clairaut's Theorem)**.** *Suppose f is defined on a disk D that contains the point (a,b). If the functions f_{xy} and f_{yx} are both continuous on D, then*

$$f_{xy}(a, b) = f_{yx}(a, b)$$

[60]

Theorem 2.2.5. *Let $I \subset \mathbb{R}$ be an open interval, and let $f : I \to \mathbb{R}$ be a function. Then the following are equivalent.*

(a) If $a, b \in I$ and $a < b$, then $f(ta + (1 - t)b) \leq tf(a) + (1 - t)f(b)$ for all $t \in [0, 1]$ (Function Is Convex)

(b) Suppose that f is differentiable. Then the function is convex if and only if $f''(x) \geq 0$ *for all* $x \in I$

Note that theorem 2.2.5 is equivalent to theorem 4.6.2 and theorem 4.6.4 in " Real Analysis" by Bloch [59].

Similarly, we have the following theorem:

Theorem 2.2.6. *Let* $I \subset \mathbb{R}$ *be an open interval, and let* $f : I \to \mathbb{R}$ *be a function. Then the following are equivalent.*

(a) If $a, b \in I$ *and* $a < b$*, then* $f(ta + (1-t)b) > tf(a) + (1-t)f(b)$ *for all* $t \in [0, 1]$ *(Function Is Concave)*

(b) Suppose that f is differentiable. Then the function is concave if and only if $f''(x) < 0$ *for all* $x \in I$

2.2.2 The Model

Assumption 1 for Utility function: *Let* $U(c, h) : (\mathbb{R}^+ \bigcup\{0\})^2 \to \mathbb{R}^+ \bigcup\{0\}$ *be a twice differentiable function that satisfies the following conditions:* $U_c > 0, U_{cc} < 0, U_h > 0, U_{hh} < 0$, *and* $U_{ch} > 0$

The above conditions ensure that utility is increasing at a decreasing rate with respect to a person's consumption c and health level h. The conditions on the first derivatives imply that the more consumption c one person has and the healthier he is, the happier he is. The assumptions about the second derivatives indicate that if two people are given the same amount of consumption, the person with an initial low level of consumptions will gain more happiness than the person initally has a high level of consumptions. The cross derivative $U_{ch} > 0$ assumes that consumption and health level are complement. If a person is healthy, then a gain in consumption brings him more happiness than to people who are less healthy.

Assumption 2 for health production function: *Let* $h(x, k, H) : \mathbb{R}^{+3} \to \mathbb{R}$ *be a twice differentiable function with respect to each variable and the second derivative with respect to each variable is continuous. Conditions:* $h_x > 0, h_k > 0, h_H > 0$ *and* $h_{xx} < 0$, $h_{HH} < 0$, $h_{kk} < 0$

The interpretation of the health production function is similar to that of the utility function. Health consists of the quantity of medicine x, exposure coefficient to the environment k, and the community's health H. An individual's health level h is increasing but at diminishing rates with respect to its medicine inputs, the exposure coefficient, and the community's health.

Recall that we want to maximize the utility function $U(c, h(x))$ subject to the constraint $c + px = I$. So rewrite $c = I - px$.

Constraint: $c+px = I$ where p is the price of x (medicine), and I is income. $c, p, x, I \in \mathbb{R}^+$. Because $c \geq 0$ and $c(x) = I - px$, $x \in [0, \frac{I}{p}]$. Similarly, $c \in [0, \frac{I}{p}]$.

Let $V : [0, \frac{I}{p}] \to \mathbb{R}^+$ be a continuous on $[0, \frac{I}{p}]$ and twice differentiate function on $(0, \frac{I}{p})$ such that

$$V(x) = U(c(x), h(x)) \tag{2.2.1}$$

We want to maximize $V(x)$ with respect to x in $[0, \frac{I}{p}]$.

Assumption 3: *When* x *is sufficiently close to* 0*, denoted* $x \to 0$*, then* $\frac{(\partial U/\partial h)(\partial h/\partial x)}{\partial U/\partial c} > p$*. When* x *is sufficiently close to* $\frac{I}{p}$*, denoted* $x \to \frac{I}{p}$*, then* $\frac{(\partial U/\partial h)(\partial h/\partial x)}{\partial U/\partial c} < p$.

This assumption implies that a person needs both medicine and consumption to be happy. When x approaches to 0, the happiness gained from an increase in medicine $\frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$ is much larger than that created by a gain in consumption c. The opposite is true when consumption c is very small, i.e. when x approaches $\frac{I}{p}$. If consumption c is scarce and medicine x is very abundant, a person would be much happier to obtain c rather than x.

Theorem 2.2.7. *Let $V(x)$ be a continuous on $[0, \frac{I}{p}]$ and twice differentiate function on $(0, \frac{I}{p})$ described in formula 2.2.1, and suppose that $V(x)$ satisfies Assumptions 1, 2, and 3. Then there exists an unique $x^* \in (0, \frac{I}{p})$ such that $V(x^*)$ is a global maximum on $[0, \frac{I}{p}]$.*

Proof. By the chain rule, we have:

$$\frac{\partial V}{\partial x} = \frac{\partial U}{\partial c}.\frac{\partial c}{\partial x} + \frac{\partial U}{\partial h}.\frac{\partial h}{\partial x} \tag{2.2.2}$$

$$= -p\frac{\partial U}{\partial c} + \frac{\partial U}{\partial h}.\frac{\partial h}{\partial x} \tag{2.2.3}$$

and

$$\frac{\partial^2 V}{\partial x^2} = p^2\frac{\partial^2 U}{\partial c^2} - p\frac{\partial^2 U}{\partial c \partial h}\frac{\partial h}{\partial x} \tag{2.2.4}$$

$$- \quad p\frac{\partial^2 U}{\partial h \partial c}(c, h) + \frac{\partial^2 U}{\partial h^2}(\frac{\partial h}{\partial x})^2 + \frac{\partial U}{\partial h}\frac{\partial^2 h}{\partial x^2} \tag{2.2.5}$$

$$< \quad 0(\text{because each component is less than } 0) \tag{2.2.6}$$

As $x \to 0$, $V_x > 0$ by Assumption 3. As $x \to \frac{I}{p}$, $V_x < 0$ by Assumption 3. Therefore, there exists $x^* \in (0, \frac{I}{p})$ such that$V_x(x^*) = 0$ by the Intermediate Value Theorem.

Since $V_{xx} < 0$ for all $x \in (0, \frac{I}{p})$, x^* is unique.

For $x_1 < x^*$ for all $x_1 \in (0, x^*)$, $V_x(x_1) > V_x(x^*) = 0$. Thus, $V(x_1) < V(x^*)$

For $x_2 > x^*$ for all $x_2 \in (x^*, \frac{I}{p})$, $V_x(x_2) < V_x(x^*) = 0$. Thus $V(x_2) < V(x^*)$

Therefore, $V(x^*)$ is the unique global maximum on $(0, \frac{I}{p})$, and x^* is also unique .

Therefore, there exists an $a > 0$ such that as $x \to 0$, $V(x) \leq V(x^*) - a$. In addition, there exists $b > 0$ such that when $x \to \frac{I}{p}$, then $V(x) \leq V(x^*) - b$

Becase $V(x)$ is a continuous function on $[0, \frac{I}{p}]$, $V(x)$ is continuous at 0 and at $\frac{I}{p}$. Hence, $V(0) = \lim_{x\to 0} V(x)$ and $V(\frac{I}{p}) = \lim_{x\to \frac{I}{p}} V(x)$ by Theorem 2.2.3. Therefore, $V(0) \leq V(x^*) - a = V(^*x)$ and $V(\frac{I}{p}) \leq V(x^*) - b < V(x^*)$.

We conclude that $V(x^*)$ is the unique global maximum of $V(x)$ on $[0, \frac{I}{p}]$

□

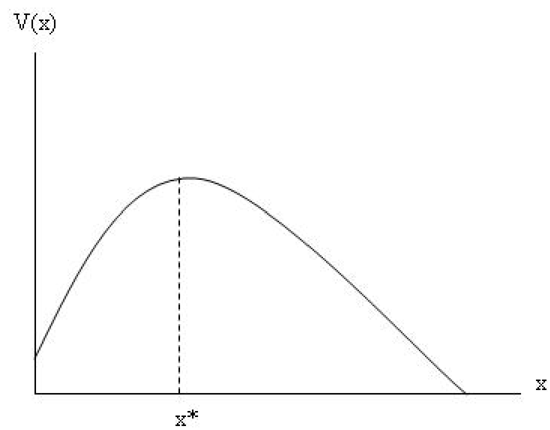


Figure 2.2.1. Example of $V(x)$

Assumption 4: *$x^*(k, I, p, H)$ is a twice differentiable function with respect to each variable. In addition, the mixed partial derivatives of x^* are continuous.*

This assumption allows us to examine properties of an individual's health input x^* when other variables change. The statement ensures that the function x^* satisfies the Clairaut's Theorem. In the functional form, this assumption is satisfied.

Assumption 5: $\frac{\partial^2 h}{\partial x \partial H} < 0$

This assumption implies that the marginal gain in health created by medicine x in a healthy environment is lower than that in a less healthy environment. This implies that a person living in a high health level community does not need to take much medicine and can still be healthy.

Theorem 2.2.8. *If Assumption 5 holds, then* $\frac{\partial x^*}{\partial H} < 0$

Proof. Recall the first derivative of V with respect to x.

$$\frac{\partial V}{\partial x} = -p\frac{\partial U}{\partial c} + \frac{\partial U}{\partial h}.\frac{\partial h}{\partial x}$$

Then

$$\frac{\partial^2 V}{\partial x \partial H} = -p\frac{\partial^2 U}{\partial c \partial h}\frac{\partial h}{\partial H} + \frac{\partial^2 U}{\partial h^2}\frac{\partial h}{\partial H}\frac{\partial h}{\partial x} + \frac{\partial U}{\partial h}\frac{\partial^2 h}{\partial x \partial H} \tag{2.2.7}$$

Equation 2.2.7 is always less than 0 because $-p\frac{\partial^2 U}{\partial c \partial h} < 0$ and $\frac{\partial^2 U}{\partial^2 h} < 0$ by Assumption 1. And $\frac{\partial^2 h}{\partial x \partial H} < 0$ by Assumption 5.

Therefore, $\frac{\partial V}{\partial x}$ decreases as H increases.

Let x_1^* and x_2^* denote the value at which the utility reaches its maximum when $H = H_1$ and $H = H_2$ respectively, with $H_1 < H_2$. Therefore, for every x, $\frac{dV}{dx}|_{H=H_1} > \frac{dV}{dx}|_{H=H_2}$ for $H_1 < H_2$. Therefore at x_1^*, $\frac{dV}{dx}|_{H=H_2} < \frac{dV}{dx}|_{H=H_1} = 0$. Thus, $x_2^* < x_1^*$.

□

As the community's health stock H increases, a person would take less medicine, i.e. x^* decreases. It is because in a healthy environment, the marginal gain by taking an additional pill is low. When the community health stock increases, there are two effects influencing an individual's health. The direct effect that H has on individual health increases the person's health, i.e. h^* increases. In addition, as H increases, x^* decreases, causing a decrease in

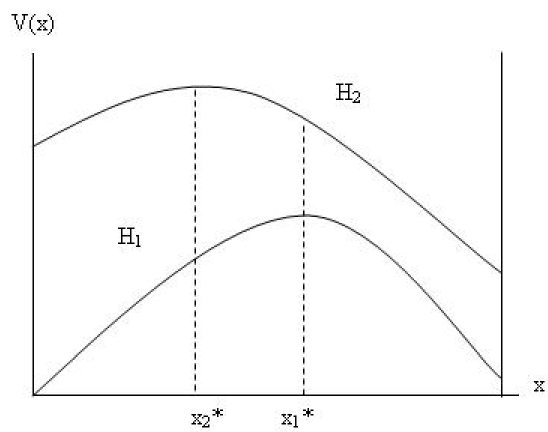


Figure 2.2.2. Example of $V(x)|_{H=H_1}$ and $V(x)|_{H=H_2}$

individual health h^*. Thus, depending on which effect is greater, an individual's health might increase or decrease as H increases. We introduce function f to study these effects.

Let f be a continuous and twice differentiable function and be defined by

$$h^* = f(x^*(I, p, k, H), k, H)$$

Consider the first derivative:

$$\frac{\partial h^*}{\partial H} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial H} + \frac{\partial f}{\partial H} \tag{2.2.8}$$

There are two effects that affect a person's health as the community's health increases. First, the direct effect of the community's health on the person's health increases this person's health. Second, the indirect effect of a decreasing medicine input decreases the person's health. Therefore, it depends on which effect is greater, this person might or might not be healthier. More specifically, if the direct effect is greater than the indirect effect, then the health level will increase if the environment health increases. However, if the indirect effect via x^* is greater than that of the community's health, then the individual's health will decrease.

An example of how an individual's health can decrease as the community health increases can be seen in the TB outbreak in New York City early 1990s. Along with the declination of TB cases in the US since the 1980s, the budget for TB prevention and treatment got smaller. However in the 1990s, immigrants from abroad once again introduced TB to New York City. Without an adequate preparation, New York City experienced a TB outbreak. In addition, there are some diseases with a cyclical spread. For instance, syphilis has cycles of 10 years in the US and 20 years in Japan [55]. Since syphilis infections occur as a result of unprotected sexual intercourse, the incidence of syphilis is strongly associated with an individual's sexual behavior. As the spread of syphilis becomes more serious, people recognize the risk of infection, and they choose safe sexual behavior. This behavior then reduces the number of syphilis infections. In a low risk environment, people are more engaged in risky sexual behaviors, and this results in high syphilis prevalence.

Next, we would like to examine the community's health equilibria. i.e how many equilibria a community can have? How stable are they? So first we need to examine the shape of an individual's health h with respect to the community's health H.

Recall that the first derivative of h with respect to H is as followed:

Examine the second derivative of h in terms of H from formula 2.2.8:

$$\frac{\partial h^*}{\partial H} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial H} + \frac{\partial f}{\partial H}$$

$$\begin{aligned} \frac{\partial^2 h^*}{\partial H^2} &= \frac{\partial^2 f}{\partial x^{*2}}\left(\frac{\partial x^*}{\partial H}\right)^2 + \frac{\partial^2 f}{\partial x^* \partial H}\frac{\partial x^*}{\partial H} + \frac{\partial f}{\partial x}\frac{\partial^2 x}{\partial H^2} \\ &+ \frac{\partial^2 f}{\partial H \partial x^*}\frac{\partial x^*}{\partial H} + \frac{\partial^2 f}{\partial H^2} \end{aligned}$$

Examine this derivative term by term:

$\frac{\partial^2 f}{\partial x^{*2}}(\frac{\partial x^*}{\partial H})^2 < 0$ because $\frac{\partial^2 f}{\partial x^{*2}} < 0$ by assumption 2.

$\frac{\partial^2 f}{\partial x^* \partial H}\frac{\partial x^*}{\partial H} > 0$ because $\frac{\partial^2 f}{\partial x^* \partial H} < 0$ by Assumption 5 and $\frac{\partial x^*}{\partial H} < 0$ by Theorem 2.2.8.

$\frac{\partial f}{\partial x^*}\frac{\partial^2 x^*}{\partial H^2}$ cannot be determined because it depends on the sign of $\frac{\partial^2 x^*}{\partial H^2}$

$\frac{\partial^2 f}{\partial H^2} < 0$ by assumption 2.

There are two effects that have an impact on the concavity of the health production function with respect to the community's health. First is the direct effect $\frac{\partial^2 f}{\partial H^2} < 0$ and $\frac{\partial^2 f}{\partial x^2} < 0$. These forces make the health production function concave if an individual's health increases as H increases. The second effect is an indirect effect via the decrease in medicine. Depending on how much the decrease of x^* depreciates the individual's health and and how much the increase of H improves his health, the health production function might be concave or convex with respect to the community's health. It also might be possible that the individual's health is concave for some values H and convex for other values of H. Theoretically, there are many possibilities that might happen when examining individual's health with respect to community's health. The followings are two examples:

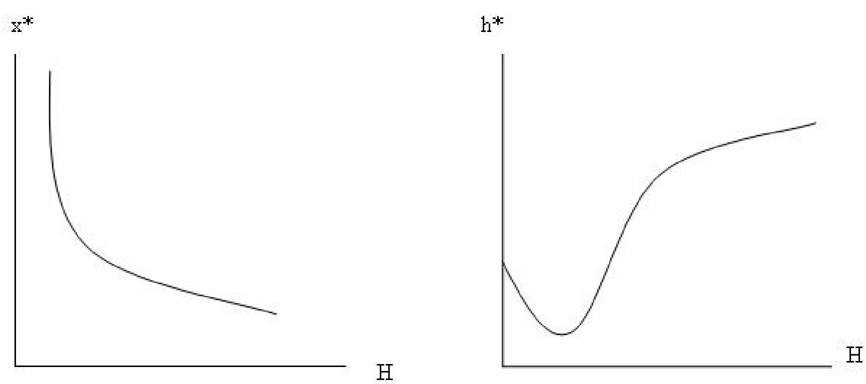


Figure 2.2.3. An example of possible behavior of health production function h with respect to the community's health H.

Figure 2.2.3 shows that x^* decreases tremendously when H is small and less so when H is large. As a result, the person's health might be as shown in figure 2.2.3. The gain from the community's health at the beginning is not enough to outweigh the large decrease of medicine. As the community's health increases more and more, this decrease in medicine accentuates and affects the indivudal's health less. Then eventually, as H is very high, the quantity of medicine does not decrease as much. As a result, the direct effect of the community's health H outweighes its indirect effect. This results in an increase in h^* with respect to H. Since we assume that $\frac{\partial^2 h}{\partial H^2} < 0$, this increase of h^* exhibits an overall diminishing returns.

In addition, it can be that the medicine consumption x^* decreases at a faster rate as H increases as shown in 2.2.4, i.e. $\frac{\partial^2 x^*}{\partial H^2} > 0$. If that is the case, then it is likely that the production function h increases at a slower rate as H increases, i.e. $\frac{\partial^2 h}{\partial H^2} < 0$. It is because the effect of medicine is not strong when H is small. When H gets larger, so is the decrease in x^* with respect to H. This can make an individual's health decreases if x^* decrease in a sufficient magnitude when H is high. On the other hand, it can also be that the direct effect still outweighs the indirect effect, i.e. if H is large enough to outweighes the decrease of x^*, then the individual's health still increases and exhibits diminishing returns.

The third senario is that the quantity of medicine decreases at a slower rate when H is small ($\frac{\partial^2 x^*}{\partial H} < 0$ when $0 < H < H_1$), then decreases at an increasing rate ($\frac{\partial^2 x^*}{\partial H} > 0$ when $H_1 < H < H_2$), and finally decreases at a slower rate when H is sufficiently large ($\frac{\partial^2 x^*}{\partial H} < 0$ when $H > H_2$). If that is the case, an individual's health level might increase with decreasing returns at first, then increase at an increasing rate, and then finally increase at a decreasing rate.

What happens if we relax assumption 5?

Recall $V_x = -p\frac{\partial U}{\partial c} + \frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$

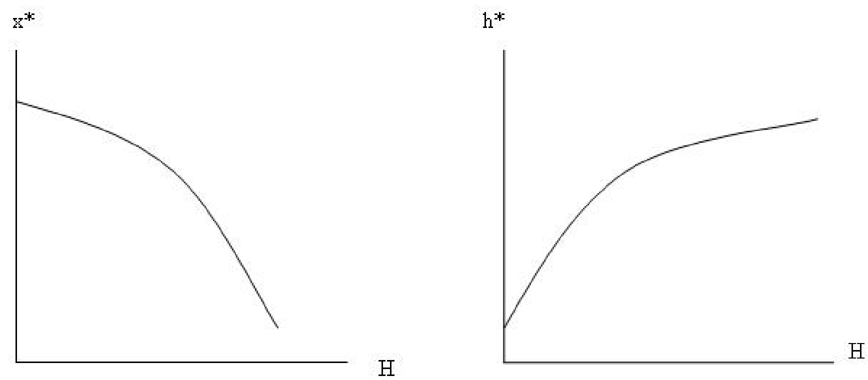


Figure 2.2.4. An example of possible behavior of health production function h with respect to the community's health H.

In general, there are two components of V_x: the change in utility via the change in consumption c and the change in utility via the change in an individual's health level. The increase in H makes h increase, which makes the marginal utility with respect to consumption c increase by Assumption 1. As a result, a person would spend more on consumption to maximize their utility. Given that their income stays constant, they would spend less money on medicine. The second component, $\frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$, is the increase in utility via the increase in health. As the community health H increases, the marginal health production function with respect to medicine will change, and so will the marginal utility with respect to health. By our assumption of the utility function (Assumption 1), the marginal utility decreases as the health level increases. In other words, $\frac{\partial U}{\partial h}$ decreases as h increases. Another aspect that needs close attention is whether or not the increase in H leads to the increase in marginal return in the health production function h with respect to x, i.e. $\frac{\partial^2 h}{\partial x \partial H}$ might be positive or negative. If medicine is more productive in a healthy community,($\frac{\partial^2 h}{\partial x \partial H} > 0$) , then $\frac{\partial h}{\partial x}$ increases as H increases. As a result, an individual might decide to spend more money on medicine. Additionally, because the total effect of medicine on utility $\frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$ might or might not outweigh the effect of consumption c on utility u, the increase in the community's health H might or might not lead to an increase in marginal utility with respect to medicine x. In other words, V_x might increase or decrease as H increases. There are three cases for $\frac{\partial^2 h}{\partial x \partial H}$

Case 1: $\frac{\partial^2 h}{\partial x \partial H} < 0$. This case is the same as Assumption 5 and we have already examined this case.

Case 2: $\frac{\partial^2 h}{\partial x \partial H} > 0$.

Note that x and H are two complementary goods.

Recall from 2.2.3 that

$$V_x = -p\frac{\partial U}{\partial c} + \frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$$

Examine the mixed partial derivatives:

$$\frac{\partial^2 V}{\partial x \partial H} = -p\frac{\partial^2 U}{\partial c \partial h}\frac{\partial h}{\partial H} + \frac{\partial^2 U}{\partial h^2}\frac{\partial h}{\partial H}\frac{\partial h}{\partial x} + \frac{\partial U}{\partial h}\frac{\partial^2 h}{\partial x \partial H}$$

Examine term by term

$-p\frac{\partial^2 U}{\partial c \partial h}\frac{\partial h}{\partial H} < 0$ because $\frac{\partial^2 U}{\partial c \partial h} > 0$ by Assumption 1.

$\frac{\partial^2 U}{\partial h^2}\frac{\partial h}{\partial H}\frac{\partial h}{\partial x} < 0$ because $\frac{\partial^2 U}{\partial h^2}$ by Assumption 1

$\frac{\partial U}{\partial h}\frac{\partial^2 h}{\partial x \partial H}$ might be positive or negative

The first term is less than 0 because as community's health increases, a person's health would increase, which makes consumption bring more utility to the person. The second term less than 0 indicates that as community's health increases, a person's health increases. This increase makes the marginal utility with respect to health smaller. The third term depends on whether or not medicine is more productive in a healthy environment. If it is, then it is more likely that individuals would choose to take more medicine. Otherwise, they would reduce their medicine consumption.

Recall from Equation 2.2.6 that $V_{xx} < 0$. Thus, V_x decreases and x increases.

However, if we assume that $\frac{\partial^2 h}{\partial x \partial H} > 0$, this means that medicine is more productive in a healthier environment. Then V_x might increase as H increases. As a result, x^* might increase. The likelihood of such increase is higher than when $\frac{\partial^2 h}{\partial x \partial H} < 0$. In other words, x^* can increase, depending on whether or not consumption or medicine effect on utility is greater. It also depends on if the decrease in marginal utility with respect to health $\frac{\partial U}{\partial h}$ outweighs the gain of the health production function as x increases. In other words, $\frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$ might increase or decrease. It is because high level of H already brings a high level of individual's health h, which leads to lower marginal utility with respect to health. In general, an individual would take more medicine in a healthy environment. As a result, it is more likely that individual's health increases at an increasing rate as H increases.

Case 3: $\frac{\partial^2 h}{\partial x \partial H} > 0$ for $H < H_1$ and $\frac{\partial^2 h}{\partial H \partial x})0$ for $H > H_1$. This case is the combination of case 1 and case 2. For any $H < H_1$, we have the same scenario as case 2. That is, we do not know whether or not an individual will respond to the increase in community's health via the increase in his medicine consumption. Or he would decrease his consumption of medicine as the environment is more healthy. The overall health in general will increase as the environmental health increases. As $H > H_1$, we know that an individual will decrease his medicine consumption because of the increase in the environmental health by case 1. It is difficult to discuss precisely the final shape of h with respect to H. It can be that as H increases, h increases at a decreasing rate, then h increases at an increasing rate, then finally h increases at a decreasing rate.

In summary, there are a lot of possibilities that an individual's health production function takes form with respect to the community's health level. An individual's health can increase as the community health increases. It can be convex, concave, or exhibit both properties.

We define the community health as an average of all individual's health, denoted $m(H) = \frac{1}{n}\sum_{i=1}^{n} h_i$. The community's health production function $m(H)$would also exhibit many shapes. It is possible that the community's health reaches two stable multiple equilibria because community health is an average of every individual's health in the community.

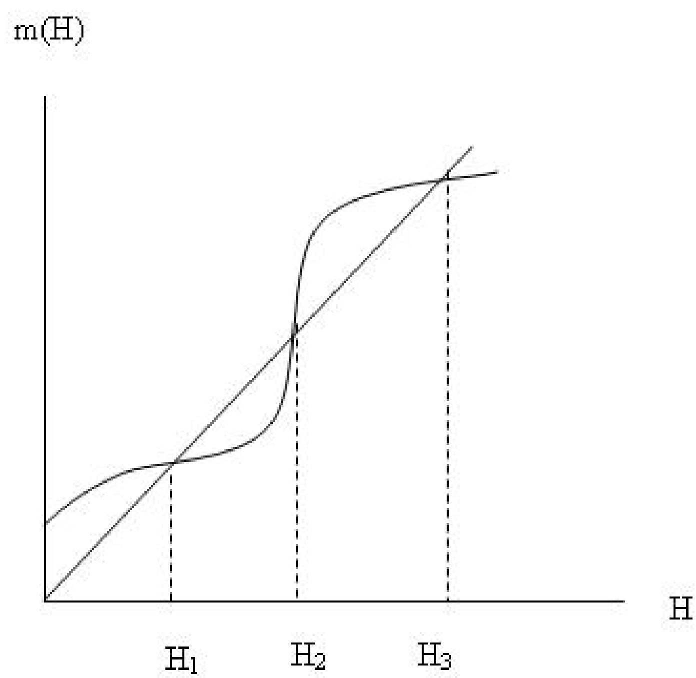


Figure 2.2.5. A possibility of multiple equilibria H.

If there are several stable equilibria, then it is possible for a community to stay in one equilibrium while the other is in another. For instance in Figure 2.2.5, when $H \rightarrow H_1$, then the community's health level will converge to H_1. And if somehow the community's health level is high enough, reaching H_3, it will reach a higher equilibrium health level $H = H_3$. This brings an issue of the reciprocal relationship between community's health and individual's health. A community that has many healthy individuals possesses a healthy environment. And in exchange, this healthy environment will have a positive feedback to its people, which makes the entire community healthier. On the other end of the spectrum, a community with many sick people will experience a less healthy environment. In exchange, this environment has a negative impact on the community's people, which results in a lower level of health for individuals. Unless there is a large change in the community of the low health level (from H_1 to H_3), every small change in community's health will revert to its stable equilibrium H_1.

If a response health function of a disease in a country ($m(H)$) has multiple equilibria, it would be in the interest of policy makers and international organizations to know where the equilibria are, so they can decide how much effort to put into a short-run disease prevention and treatment campaign. For instance, country A has high HIV/AIDS prevalence (low H_1). Policy makers and international organizations want to implement a policy to improve the situation. But how much resources is enough to reach a new sustainable outcome? If policy makers stop somewhere in between H_1 and H_2, then eventually the disease will revert back to its current situation H_1. On the other hand, if policy makers reduce the investment when the community's health level is beyond H_2 and switch back to a normal prevention and treatment scheme, then the disease's prevalence will reach its higher equilibrium H_3. As a result, knowing how many equilibria it has, where they are, and how long it takes to reach those equilibria are important questions for public health to consider. If the disease already reached its highest equilibria, then a short-run campaign will not be sustainable in the long run, unless policy makers decide to change the normal budget allocated to that disease permanently. The cyclic nature of syphilis in Japan and the US can demonstrate that money into syphilis intervention was apparently not enough for the disease to reach its new equilibrium. Every time there is a syphilis outbreak, the government increases its budget to fight the disease. But apparently they did not terminate the campagin on the right interval of H that allows syphilis to reach its new equilibrium. On the other hand, if "high" syphilis prevalence is already a maximum level or the only equilibrium that syphilis can reach given a normal budget, then a temporary intervention is not going to result in a sustainable outcome unless the government can raise the budget permanently. Note that this permanent increase does not need to be the same amount as the temporary increase.

The existence of multiple equilibria in this case solely comes from the change in concavity of the community's health production function. And this change in concavity is a result of the decision of taking more medication or not when the community's health increases. It can be because the medicine is not as effective in a low health environment as it is in a healthier environment, so a small increase in the community's health leads to a huge decrease in medicine inputs. In general, a person's health is not only affected by that person's behavior, but it is also influenced by the environmental externality. Sometimes, the externality is so large that it outweighes the individual's behavior.

Theorem 2.2.9. *: Assume that the solution x^* satisfies Assumption 4, then as p decreases, x^* increases. In other words, $\frac{\partial x^*}{\partial p} < 0$.*

Proof. Recall the first derivative of V with respect to x.

$$\frac{\partial V}{\partial x} = -p\frac{\partial U}{\partial c} + \frac{\partial U}{\partial h}.\frac{\partial h}{\partial x}$$

Then the cross derivative of $V(x)$ with respect to x and p are

$$\begin{aligned}\frac{\partial^2 V}{\partial x \partial p} &= -\frac{\partial U}{\partial c} - p\frac{\partial^2 U}{\partial c^2}\frac{\partial c}{\partial p} + \frac{\partial^2 U}{\partial h \partial c}\frac{\partial c}{\partial p}\frac{\partial h}{\partial x} \\ &= -\frac{\partial U}{\partial c} + px\frac{\partial^2 U}{\partial c^2} - x\frac{\partial^2 U}{\partial h \partial c}\frac{\partial h}{\partial x} < 0\end{aligned}$$

Follow the same proof as theorem 2.2.8, we have as p decreases, x^* increases. □

As the cost of medicine decreases, people will consume more medicine, and they will be healthier. Therefore, a government's subsidization of medicine and vaccines is one of the main strategies for alleviating the burden of infectious diseases.

Income segregation

Let us examine what happens to an individual's health as his income I increases. We assume that a change in an individual's income can only affect his health but is significant when comes to the community's health stock H.

Theorem 2.2.10. *As I increases, so is x^**

Proof. Let I_1 and I_2 be two different incomes and suppose that $I_1 < I_2$. Then $V(x)$ has a maximum at x_1^* on $[0, \frac{I_1}{p}]$ and a maximum at x_3^* on $[0, \frac{I_2}{p}]$ by the proof of Theorem 2.2.7. Recall the first derivative:

$\frac{\partial V}{\partial x} = -p\frac{\partial U}{\partial c} + \frac{\partial U}{\partial x}$

Consider the interval $[0, \frac{I_1}{p}]$ and let x_2^* be the point where $V(x)$ achieves its maximum on $[0, \frac{I_1}{p}]$. For a given x, we always have $-p\frac{\partial U}{\partial c}|_{I=I_1} < -p\frac{\partial U}{\partial c}|_{I=I_2}$ because $c_1 = I_1 - x < I_2 - x = c_2$. In addition, for a given x, $\frac{\partial U}{\partial h}\frac{\partial h}{\partial x}|_{I=I_1} = \frac{\partial U}{\partial h}\frac{\partial h}{\partial x}|_{I=I_2}$. Therefore, on the interval $[0, \frac{\partial I_1}{\partial p}]$, $\frac{\partial V}{\partial x}|_{I=I_1} < \frac{\partial V}{\partial x}|_{I=I_2}$. Follow the similar proof as that of Theorem 2.2.8, we have $x_2^* > x_1^*$

Since $[0, \frac{I_1}{p}] \subset [0, \frac{I_2}{p}]$, so $x_3^* \geq x_2^*$. As a result, $x_3^* > x_1^*$. Therefore, we prove that as income increases, so is x^*.

□

The proof can be referred to in Figure 2.2.6:

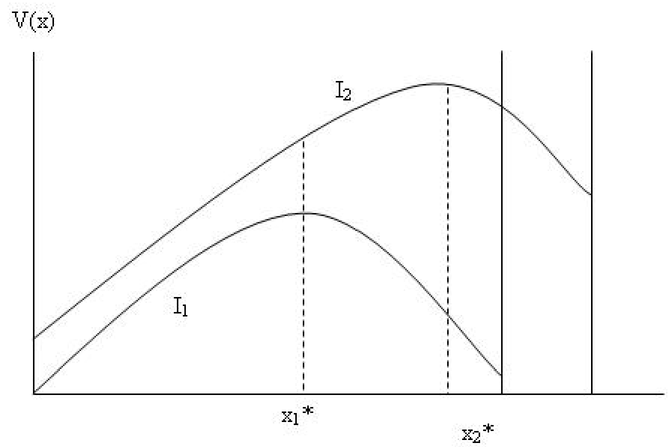


Figure 2.2.6. x^* increases when I increases from I_1 to I_2

We examine income segregation by assuming that the exposure coefficient k is associated with income I. This examination is solely from the individual's standpoint. Therefore it does not affect the macro health stock. In general, we have $h^* = f(x^*(I, p, k, H), k(I), H)$

Assumption 6: $\frac{dk}{dI} > 0$

Examine the first derivative:

$$\frac{\partial h^*}{\partial I} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I} + \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k}\frac{\partial k}{\partial I} + \frac{\partial f}{\partial k}\frac{k}{I}$$

Second derivative:

$$\begin{aligned}
\frac{\partial^2 h^*}{\partial I^2} &= \frac{\partial^2 f}{\partial x^{*2}}(\frac{\partial x^*}{\partial I})^2 + \frac{\partial^2 f}{\partial x^{*2}}\frac{\partial x^*}{\partial k}\frac{\partial k}{\partial I}\frac{\partial^2 h}{\partial x \partial k}\frac{\partial x^*}{\partial I}\frac{\partial k}{\partial I} \\
&+ \frac{\partial f}{\partial x^*}\frac{\partial^2 x^*}{\partial I \partial k}\frac{\partial k}{\partial I} + \frac{\partial f}{\partial x^*}\frac{\partial^2 x^*}{\partial I^2} \\
&+ \frac{\partial^2 f}{\partial k^2}(\frac{\partial k}{\partial I})^2 + \frac{\partial^2 f}{\partial x^* \partial k}\frac{\partial x^*}{\partial I}\frac{\partial k}{\partial I} + \frac{\partial f}{\partial k}\frac{\partial^2 k}{\partial I^2} \\
&+ \frac{\partial^2 f}{\partial k^2}(\frac{\partial k}{\partial I})^2 + \frac{\partial^2 f}{\partial x^* \partial k}\frac{\partial x^*}{\partial I}\frac{\partial k}{\partial I} + \frac{\partial f}{\partial k}\frac{\partial^2 k}{\partial I^2} \\
&+ \frac{\partial^2 f}{\partial x^{*2}}\frac{\partial x^*}{\partial I}\frac{\partial x^*}{\partial k}\frac{\partial k}{\partial I} + \frac{\partial^2 h}{\partial x^{*2}}(\frac{\partial x^*}{\partial k})^2(\frac{\partial k}{\partial I})^2 + \frac{\partial^2 h}{\partial x^* \partial k}(\frac{\partial k}{\partial I})^2\frac{\partial x^*}{\partial k} \\
&+ \frac{\partial f}{\partial x^*}\frac{\partial^2 x^*}{\partial k \partial I}\frac{\partial k}{\partial I} + \frac{\partial h}{\partial x^*}\frac{\partial^2 x^*}{\partial k^2}(\frac{\partial k}{\partial I})^2
\end{aligned}$$

Most of the terms listed above are negative, which indicates the diminishing marginal return. However, there are still some ambiguities as to whether or not the health production function is convex or concave with respect to individual's income. For instance, if medicine is more productive in a high exposure environment, a person would choose to take more medicine as the community's health increases, i.e. $\frac{\partial^2 x^*}{\partial k \partial I}$ might be greater than 0. That is, a person having high income will have a double effect. The first is the direct effect of income on health. The higher the income is, the more medicine this person can purchase. As a result, the higher the income is, the more healthy this individual is. There is also another effect via the change in the exposure coefficient. As income increases, so does the coefficient k. There are two effects on health as income increases: income effect via medication and income effect via exposure. Even if we assume that both effects are increasing at a diminishing rate, their combination effect might lead to the increasing return to scale at some point. That is, $\frac{\partial^2 h}{\partial I^2}$ might be greater than 0.

The effect of the group income on an individual's health

This subsection examines the relationship between the group income and an individual's health. It also concludes this chapter by connecting all of our previous discussions.

Let I_g be the group income, which can be thought of as an average of all individual's income. If $I_g = I$, or the group income is equal to all individual's income, then the impact of the group income on an individual health can be analyzed using the following first derivative:

$$\frac{\partial h^*}{\partial I_g} = \frac{\partial h^*}{\partial I} + \frac{\partial h^*}{\partial H}\frac{\partial H}{\partial I_g} = \frac{\partial f}{\partial I} + \frac{\partial f}{\partial k}\frac{\partial k}{\partial I} + \frac{\partial h^*}{\partial H}\frac{\partial H}{\partial I}$$

The following are three factors in which the group income impact an individual's health:

1. If $I_g = I$ and as an individual's income I increases, h^* is concave with respect to I via the direct income effect ($\frac{\partial^2 f^*}{\partial I^2} < 0$). When combining with segregation (because k is a function of I) , the function can be convex, or it can change its concavity (because $\frac{\partial h^*}{\partial I} = \frac{\partial f}{\partial I} + \frac{\partial f}{\partial k}\frac{\partial k}{\partial I}$). Note that if $I_g \neq I$, then $\frac{\partial h}{\partial I}$ plays a minimal role on $\frac{\partial h^*}{\partial I_g}$. Overall, $\frac{\partial h^*}{\partial I}$ might be postive or negative.

2. Independently, an indivudal's health h can have a convex or concave function with respect to the community health stock H, depending on the productivity of medicine in a certain environment. In addition, the community's health stock is an average of all individual's health. Thus H might be a concave, convex, or mixed function with respect to the group income.

3. As the group income I_g (this also implies that the individual's income I increases if $I_g = I$), then personal effect combines with the environment effect, which might result in a concave or convex function of h with respect to the group income I_g.

3

Functional Form

This chapter provides a more detailed and explicit discussion of the effect of income inequality and residential segregation on health via a specific functional form.

Utility function: $U_i(c_i, h_i) = c_i^{\alpha} h_i^{\beta}$ where $\alpha, \beta \in (0, 1)$.

α and β are the preference parameters of c_i and h_i respectively. The higher α is, the more this person prefers consumption goods c_i. The higher β is, the more this person likes to be healthy. α and β take values in $(0, 1)$, which represents a diminishing marginal utility with respect to each activity holding the other constant. The utility function satisfies Assumption 1.

Health production function: $h_i = (x_i + k_i H)^{\gamma}$ with $0 < \gamma < 1$.

The health production function satisfies Assumption 2 and Assumption 5.

Community's health level: $H = \frac{\sum_{i=1}^{n} h_i}{n}$. Thus, the community's health level is defined as an average of all individuals in the community.

Budget constraint: $c_i + px_i = I_i$

Our goal is to maximize utility with respect to medicine x_i and other consumption goods c_i. For now, we assume that each individual takes the disease exposure k_i and the community's health stock H as exogenous factors. Based on his budget, health production function, and utility function, an individual maximizes his utility by allocating his income to medicine or to other consumption goods.

Differentiate U_i with respect to c_i and x_i using the Lagrange multiplier λ, we get the first order condition:

$$\frac{\partial U_i}{\partial c_i} = \alpha c_i^{\alpha-1} h_i^{\beta} = \lambda \tag{3.0.1}$$

$$\frac{\partial U_i}{\partial x_i} = \beta\gamma c_i^{\alpha} h^{\beta-1}(x_i + k_i H)^{\gamma-1} = \lambda p \tag{3.0.2}$$

Dividing equation 3.0.1 with equation 3.0.2, we have

$$\frac{\beta\gamma}{\alpha}\frac{c_i}{h_i}(x_i + k_i H)^{\gamma-1} = p$$

$$\frac{\beta\gamma}{\alpha}\frac{c_i}{x_i + k_i H} = p$$

Then, we can write c_i in terms of x_i

$$c_i = \frac{\alpha}{\beta\gamma} p(x_i + k_i H) \tag{3.0.3}$$

Substituting equation 3.0.3 to the budget constraint $c_i + px_i = I_i$, we get

$$\frac{\alpha}{\beta\gamma} p(x_i + k_i H) + px_i = I_i$$

$$px_i(\frac{\alpha}{\beta\gamma} + 1) = I_i - \frac{\alpha}{\beta\gamma} pk_i H$$

Solve for x_i^* that maximizes the level of utility

$$x_i^* = \frac{I_i}{p(\frac{\alpha}{\beta\gamma}+1)} - \frac{\frac{\alpha}{\beta\gamma}k_iH}{\frac{\alpha}{\beta\gamma}+1} = \frac{1}{\alpha+\beta\gamma}\left[\beta\gamma\frac{I_i}{p} - \alpha k_i H\right] \tag{3.0.4}$$

In addition, we restrict x_i and c_i to be between 0 and $\frac{I}{p}$, which is equivalent to the two following equations

$$0 \leq \frac{1}{\alpha+\beta\gamma}\left[\beta\gamma\frac{I_i}{p} - \alpha k_i H\right] \tag{3.0.5}$$

$$\frac{1}{\alpha+\beta\gamma}\left[\beta\gamma\frac{I_i}{p} - \alpha k_i H\right] \leq \frac{I_i}{p} \tag{3.0.6}$$

The formula 3.0.6 is always true, and the formula 3.0.5 is satisfied when $\frac{\alpha k_i H}{\beta\gamma} \leq \frac{I_i}{p}$. Therefore,

$$H \leq \frac{I_i\beta\gamma}{pk_i\alpha} \tag{3.0.7}$$

Equation 3.0.4 implies that as an individual's income I_i increases, that person will purchase a higher level of medicine to improve his health. High income I_i leads to high medication x_i* is also a result of Theorem 2.2.10. In addition, as a community's health level H increases, the level of medicine x_i^* decreases. This is because as the community's health increases, this person does not need to take as much medicine to reach the same level of health prior to the increase of the health stock. The same interpretation applies to the health exposure coefficient k_i. This result is consistent with our result of the general form in the previous chapter.

Solving for an individual's health maximization problem, we get

$$h_i^* = \left(\frac{I_i}{p(\frac{\alpha}{\beta\gamma}+1)} - \frac{\frac{\alpha}{\beta\gamma}k_i H}{\frac{\alpha}{\beta\gamma}+1} + k_i H \right)^\gamma \tag{3.0.8}$$

$$= \left[\frac{I_i}{p(\frac{\alpha}{\beta\gamma}+1)} + \frac{k_i H}{\frac{\alpha}{\beta\gamma}+1} \right]^\gamma = \left[\frac{\beta\gamma}{\alpha+\beta\gamma} \left(\frac{I_i}{p} + k_i H \right) \right]^\gamma \tag{3.0.9}$$

From equation 3.0.9, we get $\frac{\partial h_i^*}{\partial H} > 0$ and $\frac{\partial^2 h_i^*}{\partial H^2} < 0$. Therefore, function h_i^* with respect to H is increasing and concave. Even though this person will spend less money on medicine, the direct effect (increase in H) compensates the indirect effect (decrease in x^*). The final result is that as the community health increases, and the individual health increases but at a slower rate with respect to the community's health level.

Now, let us take an average of all individual's health.

$$s(H) = \frac{1}{n}\sum_{n=1}^{\infty} h_i^* = \frac{1}{n}\sum_{n=1}^{\infty} \left[\frac{\beta\gamma}{\alpha+\beta\gamma} \left(\frac{I_i}{p} + k_i H \right) \right]^\gamma \tag{3.0.10}$$

Our goal is to find $H^* \in (0, \infty)$ such that $s(H) = H$

Note that s is a function of I_i, p_i, k_i, and H. From now on, the notation $s(H)$ denotes function s with respect to H when holding other variables constant.

The rest of the chapter examines five different cases of income inequality and segregation.

3.1 Everyone is identical in both income and exposure

In this section, we discuss our base case where everyone in the community is the same. That is, $I_i = I_0$ and $k_i = k_0$ for all person i in the community. In this case, we treat I_0 and k_0 as random variables

It follows that

$$s(H) = \frac{1}{n}\sum_{n=1}^{\infty} h_i^* = h_0 = \left[\frac{\beta\gamma}{\alpha+\beta\gamma} \left(\frac{I_0}{p} + k_0 H \right) \right]^\gamma \tag{3.1.1}$$

Because everyone is identical, the community's health stock equals to each individual health.

Assumption 7: $\frac{I_i\beta\gamma}{pk_i\alpha} > 1$

This assumption indicates that an individual's income is high enough and the positive externality coefficient k is low enough, so that people need to consume medicine.

Theorem 3.1.1. *Let* $l : [0, \frac{I_0}{p}] \to \mathbb{R}$ *be a function defined by* $l(H) = s(H) - H$*, with* $s(H)$ *defined in formula* 3.1.1 *and* I_0 *satisfies Assumption 7. Then* $l(H) = 0$ *has a unique and stable equilibrium*

Proof. **a. Proof the existence of equilibrium**

First, we have $l(0) = s(0) - 0 > 0$

Then

$$l\left(\frac{I_0\beta\gamma}{pk_0\alpha}\right) = \left(\frac{I_0\beta\gamma}{p\alpha}\right)^\gamma - \frac{I_0\beta\gamma}{pk_i\alpha} \tag{3.1.2}$$

$$< \left(\frac{I\beta\gamma}{pk_0\alpha}\right)^\gamma - \frac{I\beta\gamma}{pk_0\alpha} < 0 \tag{3.1.3}$$

$$\tag{3.1.4}$$

Because of Assumption 7, then $\frac{I_0\beta\gamma}{pk_0\alpha} > 1$. Formula 3.1.4 less than 0 is always true.

Applying the Intermediate Value Theorem to the twice differentiable function $l(H)$ on interval $(0, \frac{I_0\beta\gamma}{pk_0\alpha})$, there exists a H^* on the interval $(0, \frac{I_0\beta\gamma}{pk_0\alpha})$ such that $l(H^*) = 0$.

b/ Let H_0 be the smallest solution for $l(H) = 0$. Then for $H \in (0, H_0)$, we have $g(H) = s(H) - H > 0$. Let $\epsilon > 0$, then $s(H_0 - \epsilon) > H_0 - \epsilon$. Then

$$s'(H_0) = \lim_{\epsilon \to 0^+} \frac{s(H_0) - s(H_0 - \epsilon)}{\epsilon} \leq \frac{H_0 - H_0 + \epsilon}{\epsilon} = 1$$

.

Therefore, H_0 is a stable equilibrium

c/ We will now prove that H_0 is the unique equilibrium by contradiction

Because $s'(H_0) < 1$ and $s''(H) < 0$, then for $H \in (H_0, \frac{I_0\beta\gamma}{pk_0\alpha})$, we have $s'(H) < 1$.

Assume that there exists $H_1 \in (H_0, \frac{I_0}{p})$ such that $f(H_1) = 0$. Then by the Mean Value Theorem, there exists $H_2 \in (H_0, H_1)$ such that $s'(H_2) = \frac{s(H_1)-s(H_0)}{H_1-H_0} = \frac{H_1-H_0}{H_1-H_0} = 1$, contradiction.

Therefore, H_0 is the unique stable equilibrium on $(0, \frac{I_0\beta\gamma}{pk_0\alpha})$

In addition, $l(0) \neq 0$ and $l(\frac{I_0\beta\gamma}{pk_0\alpha}) \neq 0$. We conclude that H_0 is the unique stable equilibrium on $[0, \frac{I_0\beta\gamma}{pk_0\alpha}]$

□

This community has a unique and stable equilibrium where everyone consume a positive quantity of medicine. This conclusion comes from Assumption 7 when we assume an individual's income is high enough to purchase some postive quantity of medicine. However, if income is too low and medicine is too expensive, people might choose not to take medicine, i.e. $x^* = 0$. In addition, if exposure to postive externality is too high, i.e. high k_0, then people will also decide not to take medicine. Then the effect of community's health on the individual's is greater than the effect in the case where people choose to take a positive quantity of medicine. The function is no longer smooth. We demonstrate this scenario in Figure 3.1.1 but are not going to provide an in-depth analysis in this project.

3.2 People have different incomes: Introduce the existence of income inequality

Now we assume that everyone has the same exposure k, but there are two different income groups I_1 and I_2 with the proportion of q and $1 - q$ respectively ($I_1 < I_2$). We also have $qI_1 + (1 - q)I_2 = I_0$. We treat I_1 and I_2 as two random variables and treat I_0 is treated as a constant term.

By creating two income groups, we essentially create a unequal community. There are different ways to alleviate inequality. Income inequality can be changed by changing the

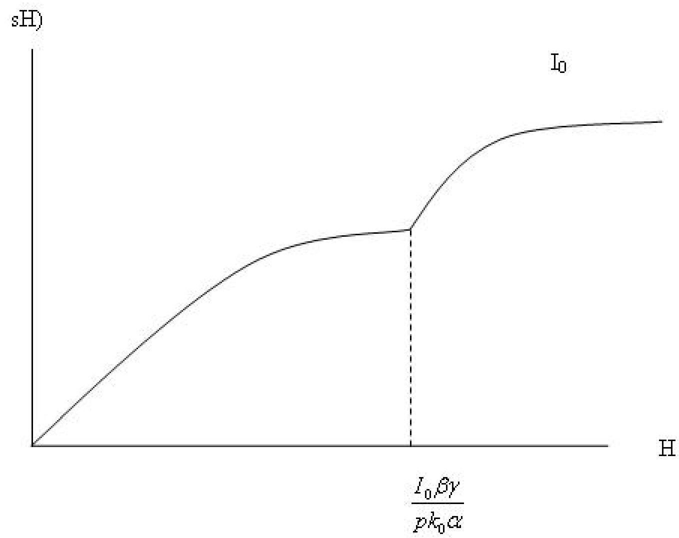


Figure 3.1.1. Community health response function

proportion of the poor (q) or the absolute income of the poor (I_1). If we keep the proportion q constant, then by increasing income I_1 of the poor group, we increase equality. On the other hand, if we keep income of the poor I_1 constant, then a decrease in the proportion q leads to a decrease in income inequality.

3.2.1 Reducing income inequality by providing all poor people with the same extra income.

This section examines how alleviating income inequality by increasing income of all poor people with the same amount affect the community's and an individual's infectious disease transmission. To analyze this scheme, we increase I_1. As I_1 increases, inequality decreases when $I_1 < I_2$. We hold the mean constant: $qI_1 + (1-q)I_2 = I_0$ constant. Thus as I_1 increases, $I_2 = \frac{I_0 - qI_1}{1-q}$ decreases.

Then, $s(H)$ is simplified to

$s(H) = qh_1 + (1-q)h_2$

For simplicity when analyzing income inequality, we define

$$m(H) = q(I_1 + kH)^\gamma + (1-q)(I_2 + kH)^\gamma = q(I_1 + kH)^\gamma + (1-q)\left(\frac{I_0 - qI_1}{1-q} + kH\right)^\gamma \quad (3.2.1)$$

Function $m(H)$ with respect to H has the same properties as function $s(H)$

Theorem 3.2.1. *As the group income I_1 decreases, H^* increases*

Proof. Let us examine the first derivative of $m(H)$ with respect to the group income I_1

$$\begin{aligned} \frac{\partial m(H)}{\partial I_1} &= q\gamma(I_1 + kH)^{\gamma-1} + (1-q)\gamma\frac{-q}{1-q}\left(\frac{I_0 - qI_1}{1-q} + kH\right)^{\gamma-1} \\ &= q\gamma(I_1 + kH)^{\gamma-1} - q\gamma(\frac{I_0 - qI_1}{1-q} + kH)^{\gamma-1} > 0(\text{since } I_1 < I_2 \text{ and } \gamma < 1) \end{aligned}$$

As I_1 increases, $m(H)$ shifts upwards for a given H, resulting in a higher level of equilibrium H^*. Thus, income equality results in a higher level of community's health. Notice that if $I_1 > I_2$, an increase in I_1 is equivalent to an increase in income inequality. In addition, an increase in I_1 will lead to a lower overall health stock H. Therefore, the maximum health stock is reached when $I_1 = I_2 = I_0$. We conclude that as income inequality decreases, i.e. I_1 increases, the community's equilibrium health H^* increases

□

Let us examine the effect of income redistribution on an individual's health and the community's health stock. Two variables that we will discuss in the individual's health function are the group income I_1 and the equilibrium community's health stock H^*. In addition, recall that H^* is a fuction of I_i, p, and k_i

$h_1 = f(I_1, H^*)$ and $h_2 = f(I_2, H^*)$

$$\frac{\partial h_1}{\partial I_1} = \frac{\partial f}{\partial I_1} + \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_1}$$

.

$$\frac{\partial h_2}{\partial I_2} = \frac{\partial f}{\partial I_2} + \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_2}$$

The community health after redistribution

$$\begin{aligned}\frac{\partial h_2}{\partial I_1} &= \frac{\partial f}{\partial I_1}\frac{\partial I_2}{\partial I_1} + \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_2}\frac{\partial I_2}{\partial I_1}\\ &= \frac{-q}{1-q}(\frac{\partial f}{\partial I_2} + \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_2})\end{aligned}$$

$$\begin{aligned}\frac{\partial H^*}{\partial I_1} &= q(\frac{\partial f}{\partial I_1} + \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_1}) + (1-q)(\frac{-q}{1-q}(\frac{\partial f}{\partial I_2} + \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_2}))\\ &= q(\frac{\partial f}{\partial I_1} - \frac{\partial f}{\partial I_2}) + q(\frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_1} - \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_2})\end{aligned}$$

From the above equation, there are two effects that I_1 has on H^*: the difference between the direct gain of the poor and the rich when their incomes increase, $q(\frac{\partial f}{\partial I_1} - \frac{\partial f}{\partial I_2})$, and the indirect effect of the poor and the rich via the increase in the common health stock when their incomes increase, $q(\frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_1} - \frac{\partial f}{\partial H^*}\frac{\partial H^*}{\partial I_2})$. As the above proves, as I_1, H^* increases. Thus, the total effect of I_1 on H^* is $\frac{\partial H^*}{\partial I_1} > 0$. In addition, the direct effect of one's income with respect to his health is increasing at a decreasing rate, i.e. $\frac{\partial h_1}{\partial I_1} - \frac{\partial h_2}{\partial I_2} > 0$. The the indirect effect of the poor is likely to be greater than that of the rich because the individual's health is concave with respect to income I_1. That is, $(\frac{\partial h_1}{\partial H^*}\frac{\partial H^*}{\partial I_1} - \frac{\partial h_2}{\partial H^*}\frac{\partial H^*}{\partial I_2}) > 0$. In other words, the poor gain more than the rich in both ways: directly via his gain in income, and indirectly via his gain as the common health stock increases. In addition, as q increases, the total effect $\frac{\partial H^*}{\partial I_1}$ also increases. Therefore, a community that has a large poor population will be more beneficial from redistribution than one that has a small portion of poor people. Holding the mean income of both community constant, as q is greater, so is I_2. This community has a small and very rich population, i.e. low $1 - q$ and high I_2. As a result, taking away a large part of the rich's income would not hurt them much. This extra income will benefit the poor to a great extent. This analysis demonstrates that in a country where there are many poor people, it is ideal to improve the poor's life because there are two gains associated with such policy. First is their direct income effect on the poor's health, and second is their contribution to the health stock and creates a positive externality as their health improved.

Notice that this argument does not hold if an individual's health is a convex function with respect to income. Then the gain of the poor is not enough to compensate for the loss of the rich, which results in a worse community's health.

An individual's health after redistribution

A person's gain in health when his income increases via a direct effect $\frac{\partial h_1}{\partial I_1}$ and an indirect effect $\frac{\partial h_1}{\partial H^*}\frac{\partial H^*}{\partial I_1}$. The increase in income affects an individual directly via the quantity of medicine he can buy. Because he is now healthier, other people around him get healthier, which increase the community's stock H^*. This increase in turn increases his health. On the one hand, the rich's goes down directly via the decrease of his income after redistribution. Via the direct effect, the rich is now healthier because the community's health stock increases. Depending on which effect is greater, the rich's health might or might not decrease. The case where the rich's health goes up implies the strong version of income inequality that discussed in Chapter 1, where everyone benefits after redistribution. Imagine that there are a lot of poor and sick people in the community. The rich who can afford medicine would consume a large amount medicine to protect themselves from getting sick (because H^* is low now). However, if instead of putting this money to themselves, the rich decided to invest in the poor's health. By giving away the money, the rich decreases their capacity to buy medicine, so their health declines. Now the poor can afford medicine and improve their health. By increasing the poor's health, the community health stock increases, which creates a positive externality to the rich. So now, even when the rich does not spend that much money to buy medicine, they can enjoy an externality from the poor's gain in health. The degree of this gain also depends on how much an unit of health is increased by an unit of income for the poor. If this gain is high, the poor gets a large gain of health from a small gain in income.

3.2.2 Income inequality can be decreased by decreasing proportion of the poor

Another way to reduce income inequality is to decrease the number of poor people, which is slightly different from as previously discussed, subsection 3.2.1, where we increase income of every poor person in the community and keep the proportion between the two groups the same. Specifically, this subsection discusses the case where a small portion of the poor has a large amount of extra income, whereas everyone gain a small amount of money in subsection 3.2.1. Note that when $q = 0$ is the same as when $I_1 = I_2 = I_0$, because the two scenarios are perfect equality. While the final results are the same, the two processes of reaching perfect equality yield different implications. We will now examine this scheme of improving equality affects a person's health, given that an individual's income stays constant and only the porportion of the poor, q, changes.

We will use the Gini coefficient as our measure for income inequality. Recall that the Gini coefficient is the ratio between the area above the Lorenz curve and the equality triangle, i.e. the area of triangle ABC over the area of AEC. We will show that as the proportion of the poor decreases, income inequality in the community decreases, i.e. the Gini coefficient decreases. We are holding I_1 and the average I_0 constant and only varying I_2.

If the community is perfectly equal, then the accumulated income is

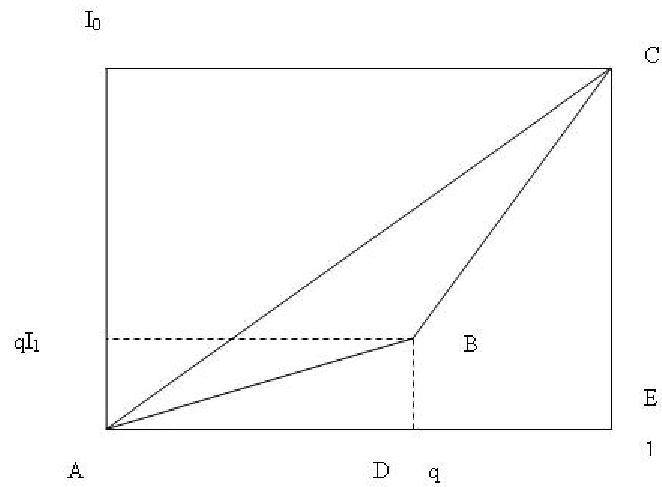


Figure 3.2.1. The Gini Coefficient, $\frac{\text{ABC}}{\text{AEC}}$

$$\text{ACE} = \frac{1.1.I_0}{2} = \frac{I_0}{2}$$

In the case where there are two income groups, the accumulated income of the poor is

$$\text{ADB} = \frac{q.q.I_1}{2} = \frac{q^2 I_1}{2}$$

The accumulated income of the rich is

$$\text{DBCE} = \frac{(qI_1 + I_0)(1-q)}{2} = \frac{q(1-q)I_1 - (1-q)I_0}{2}$$

.

Then the Gini coefficient is as followed:

$$G = \frac{\text{AEC- ADB-DBCE}}{ACE} = \frac{I_0 - q^2 I_1 - q(1-q)I_1 - (1-q)I_0}{2.I_0/2} = \frac{qI_0 - qI_1}{I_0}$$

Then $\frac{dG}{dq} = \frac{I_0 - I_1}{I_0}$. Thus, as q decreases, so does the income inequality.

Let us examines what happen to the community's health as income inequality decreases as q decreases

Theorem 3.2.2. *If q decreases, the equilibrium community's health stock H^* increases*

Recall from formula 3.2.1 that

$$m(H) = q(I_1 + kH)^\gamma + (1-q)(\frac{I_0 - qI_1}{1-q} + kH)^\gamma$$

Then the first derivative of $m(H)$ with respect to the proportion q is as followed:

$$\begin{aligned} \frac{\partial m(H)}{\partial q} &= (I_1 + kH)^\gamma - (\frac{I_0 - qI_1}{1-q} + kH)^\gamma + (1-q)\gamma[\frac{I_0 - qI_1}{1-q} + kH]^{\gamma-1}.\frac{-I_1(1-q) - (-1)(I_0 - qI_1)}{(1-q)^2} \\ &= (I_1 + kH)^\gamma - (I_2 + kH)^\gamma + \gamma(I_2 + kH)^{\gamma-1}\frac{I_0 - I_1}{1-q} \\ &= (I_1 + kH)^\gamma - (I_2 + kH)^\gamma + \gamma(I_2 + kH)^{\gamma-1}(I_2 - I_1) \end{aligned}$$

Because $I_2 = \frac{I_0 - qI_1}{1-q}$, then $\frac{dI_2}{dq} = \frac{I_0 - I_1}{(1-q)^2} > 0$. Thus, the income of the rich group increases as q decreases. Factors that affect the overall community's health are an increase in income from the small portion of the poor (I_1 increases), a decrease in income of the rich I_2 decreases, and the increase portion of the high income group ($1-q$ increases). $(I_1+kH)^\gamma - (I_2+kH)^\gamma$ is the difference between the health level of the poor and the health level of the rich. In addition, $\gamma(I_2 + kH)^{\gamma-1}(I_2 - I_1)$ is the gain of the community as the proportion of high income increases. Because the health production is increasing and concave with respect to income, we expect that the gain of the poor is large enough to outweigh the loss of the rich, shifting $m(H)$ upward as the proportion of the poor q decreases, i.e. $\frac{\partial m}{\partial q} < 0$. Then the overall community health increases as the income inequality decreases.

Figure 3.2.2 shows that as q decreases, $m(H)$ shifts upward, resulting in a higher equilibrium community H^*. This results in a higher community health's stock. As the community health stock H^* increases, the individual's health also increases. Therefore, a person who

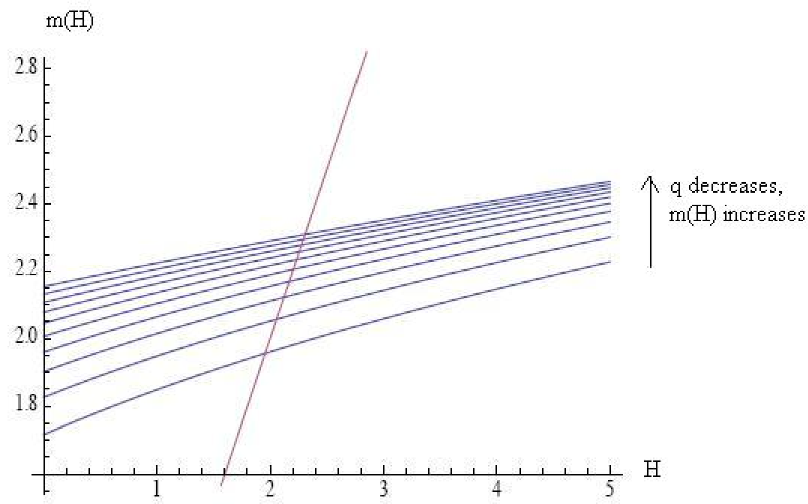


Figure 3.2.2. As q decreases, m(H) increases

lives in a more equal community is healthier than his counterpart living in less equal community.

If income equality is reached by decreasing the proportion of the poor, instead of increasing income of the entire low-income community, a person's health is concave with respect to his income. It is because his increase in income is too small to affect the community's health.

Let us compare this scheme of improving equality with the previous scheme where the same amount of money is shared by everyone. The community is better off with the scheme in subsection 3.2.1 than this scheme because we assume that an individual's health production function is increasing and concave with respect to income. As a result, giving a small amount to all poor people will result in better health for the whole community than giving a large amount to a small portion and leaving the rest poor. As income increases, its effect on health diminishes. While the final outcomes of two schemes are the same, the processes of reaching equality are different.

However, the argument does not hold if the concavity of the individual's health with respect to income is different, i.e. it might be convex or mixed between concave and convex. In that situation, the government should give large amount money to a small portion of the poor population rather than spreading it thinly among all. As that small poor portion is given money, they become a median income group. The gain of this new median income group is greater than that of the low income group because the health production function with respect to income is convex on that income interval. As a result, the community health stock would increase if extra money is given to this new median income group, instead of to the low income people.

3.3 Individual's health production function with respect to the change in group income

We know that an individual's health exhibits an increasing at a decreasing rate with respect to his individual's health. However, it might be the case that as group income increases and income inequality decreases, i.e. I_1, he has a double positive effect: his direct income effect and his indirect effect via the externality from the community's health. Community's health and income might independently exhibit marginal diminishing returns. But their combination might be convex or concave.

First, let us examine the effect of I_1 on the equilibrium community's health H^*

Since $m(H)$ is equivalent to $s(H)$, then we can write H^* in terms of I_1, k, and H^* for simplicity.

$$H^* = q(I_1 + kH^*)^\gamma + (1-q)(\frac{I_0 - qI_1}{1-q} + kH^*)^\gamma$$

The first derivative of H^* with respect to income I_1

$$
\begin{aligned}
\frac{\partial H^*}{\partial I_1} &= q\gamma(I_1+kH^*)^{\gamma-1} + qk\gamma(I_1+kH^*)^{\gamma-1}\frac{\partial H^*}{\partial I_1} \\
&- (1-q).\gamma.\frac{q}{1-q}(\frac{I_0-qI_1}{1-q}+kH^*)^{\gamma-1} + (1-q).\gamma k(\frac{I_0-qI_1}{1-q}+kH^*)^{\gamma-1}\frac{\partial H^*}{\partial I_1}
\end{aligned}
$$

Then the second derivative becomes

$$
\frac{\partial H^*}{\partial I_1} = \frac{q\gamma(I_1+kH^*)^{\gamma-1} - q\gamma(\frac{I_0-qI_1}{1-q}+kH^*)^{\gamma-1}}{1-qk\gamma(I_1+kH^*)^{\gamma-1} - (1-q).\gamma k(\frac{I_0-qI_1}{1-q}+kH^*)^{\gamma-1}}
$$

Since $I_2 > I_1$, the numerator is greater than 0. In addition, $\frac{\partial H^*}{\partial I_1} > 0$ by Theorem 3.2.1, then the denominator is greater than 0. In other words,

$$
1-qk\gamma(I_1+kH^*)^{\gamma-1} - (1-q).\gamma k\left(\frac{I_0-qI_1}{1-q}+kH^*\right)^{\gamma-1} > 0 \qquad (3.3.1)
$$

Consider the second derivative:

$$
\begin{aligned}
\frac{\partial^2 H^*}{\partial I_1^2} &= q\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2} + qk\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\
&+ qk\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} + qk^2\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2}\left(\frac{\partial H^*}{\partial I_1}\right)^2 \\
&+ qk\gamma(I_1+kH^*)^{\gamma-1}\frac{\partial^2 H^*}{\partial I_1^2} \\
&+ q\gamma(\gamma-1)\left(\frac{q}{1-q}\right)\left(\frac{I_0-qI_1}{1-q}+kH^*\right)^{\gamma-2} - qk\gamma(\gamma-1)\left(\frac{I_0-qI_1}{1-q}+kH^*\right)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\
&- qk\gamma(\gamma-1)\left(\frac{I_0-qI_1}{1-q}+kH^*\right)^{\gamma-2}\frac{\partial H^*}{\partial I_1} + (1-q)\gamma(\gamma-1)k^2\left(\frac{I_0-qI_1}{1-q}+kH^*\right)\left(\frac{\partial H^*}{\partial I_1}\right)^2 \\
&+ (1-q)\gamma k\left(\frac{I_0-qI_1}{1-q}+kH^*\right)^{\gamma-1}\frac{\partial^2 H^*}{\partial I_1}
\end{aligned}
$$

It follows that

$$\begin{aligned}
\frac{\partial^2 H^*}{\partial I_1^2} &\times \left(1 - qk\gamma(I_1 + kH^*)^{\gamma-1} - (1-q).\gamma k(\frac{I_0 - qI_1}{1-q} + kH^*)^{\gamma-1}\right) \\
&= q\gamma(\gamma-1)(I_1 + kH^*)^{\gamma-2} + qk\gamma(\gamma-1)(I_1 + kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\
&+ qk\gamma(\gamma-1)(I_1 + kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\
&+ qk^2\gamma(\gamma-1)(I_1 + kH^*)^{\gamma-2}(\frac{\partial H^*}{\partial I_1})^2 \\
&+ q\gamma(\gamma-1)(\frac{q}{1-q})(\frac{I_0 - qI_1}{1-q} - kH^*)^{\gamma-2} \\
&- qk\gamma(\gamma-1)(\frac{I_0 - qI_1}{1-q} + kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\
&- qk\gamma(\gamma-1)(\frac{I_0 - qI_1}{1-q} + kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\
&+ (1-q)\gamma(\gamma-1)k^2(\frac{I_0 - qI_1}{1-q} + kH^*)(\frac{\partial H^*}{\partial I_1})^2
\end{aligned}$$

Since $I_1 > I_2$, the right hand side is less than 0. We also have $1 - qk\gamma(I_1 + kH^*)^{\gamma-1} - (1-q).\gamma k(\frac{I_0 - qI_1}{1-q} + kH^*)^{\gamma-1} > 0$ by equation 3.3.1 As a result, the second derivative is less than 0. In other words, the function of equilibrium community health with respect to income increases is a concave function.

Now let us examine how the individual's health changes as the group income I_1 increases via the direct increase in income and the indirect change of community health.

$$h_1^{**} = (I_1 + kH^*)^{\gamma}$$

The first derivative of h_1^{**} with respect to the group income I_1 becomes

$$\frac{\partial h^{**}}{\partial I_1} = \gamma(I_1 + kH^*)^{\gamma-1} + k\gamma(I_1 + kH^*)^{\gamma-1}\frac{\partial H^*}{\partial I_1}$$

Its second derivative becomes

$$\begin{aligned}\frac{\partial^2 h_1^{**}}{\partial I_1^2} &= \gamma(\gamma-1)(I_1+kH^*)^{\gamma-2} + k\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} \\ &+ k\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2}\frac{\partial H^*}{\partial I_1} + k^2\gamma(\gamma-1)(I_1+kH^*)^{\gamma-2}\frac{\partial^2 H^*}{\partial I_1^2}\end{aligned}$$

Every term except for the last term is negative because H^* is concave with respect to I_1. This term makes the individual's health production function with respect to the group income less concave. In some situation, the function might turn to a convex function. But the health function stays concave in this specific functional form.

3.4 The effect of exposure

Holding income constant, we now change exposure coefficient k_i such that $qk_1+(1-q)k_2 = k$ is a constant. We have $m(k_i)$ is an increasing and concave function with respect to the eposure coefficient, which is analogous to the income. Following the same argument as in the previous discussion when we vary income, we have the case when variation of exposure will lead to a lower health level than in the case where the exposure coefficient is the same. The idea here is the diminishing marginal utility of the health stock. The higher the exposure coefficient is, the slower it has an effect on an individual's health level. Therefore, when we take away from high exposure people and give it to low exposure people, the gain of the original low exposure will be large enough to compensate the loss of the high exposure. Now we want to examine the individual's health level after the redistribution of exposure. Similar to the argument of income, we expect that the health level of the low exposure would increase after redistribution via a direct effect k_1 and an indirect effect through the increase of H^*. The old high exposure group also experiences two contradictory effects: the decrease of exposure k_2 would decrease the level of health h_2, and the increase in H^* would increase the individual's health level h_2.

3.4.1 Inequality and exposure are independent

Both income I_i and exposure k_i independely vary. Let I_1 and I_2 be 2 values of I_i with the proportion of q_1 and $1-q_1$ respectively. Let k_1 and k_2 be 2 values of k_i with probabilities of q_2 and $1-q_2$ respectively. Table 3.4.1 presents the probability of all possible outcomes:

k & I	I_1	I_2
k_1	q_1q_2	$(1-q_1)q_2$
k_2	$q_1(1-q_2)$	$(1-q_1)(1-q_2)$

Table 3.4.1. Interaction between I_i and k_i

$$\begin{aligned}
m(I_i,k_i) &= q_1q_2(I_1+k_1H)^\gamma+(1-q_1)q_2(I_2+k_1H)^\gamma+\\
&+ q_1(1-q_2)(I_1+k_2H)^\gamma+(1-q_1)(1-q_2)(I_2+k_2H)^\gamma\\
&= q_2[q_1(I_1+k_1H)^\gamma+(1-q_1)(I_2+k_1H)^\gamma]+\\
&+ (1-q_2)[q_1(I_1+k_2H)^\gamma+(1-q_1)(I_2+k_2H)^\gamma]\\
&< q_2g(I_0,k_1)+(1-q_2)g(I_0,k_2)(\text{when only } k_i \text{ varies, } I_i \text{ is identical by Theorem 2.2.6})\\
&< g(I_0,k_0)(\text{when both income and epxosure are identical for everyone})
\end{aligned}$$

Thus, the income level is higher in the case when both income and exposure are constant than in the cases when only one variable varies, which is lower than when everyone has identical income and disease exposure. In this case, comparing to the case where everyone is intergrated and has the same income, segregation and inequality lead to a low level of health. It is because it carries a double negative in two dimensions: spatial and income. As the community's health decreases, an individual's health also decreases.

3.5 Inequality and exposure are correlated (the case of segregation)

Consider I_i and k_i as two random variables. I_i takes two values I_1 and I_2 with $I_1 < I_2$. In addition, k_i takes two values k_1 and k_2. We assume that $\text{cov}(I_i,k_i) \neq 0$. That is, poor people I_1 tend to have high exposure k_1, and rich people I_2 tend to have low exposure k_2. Let r be the probability that this event more likely to happen than in the case when income and exposure are independent. Table 3.5 presents the probabilities of all possible outcomes:

k & I	I_1	I_2
k_1	$q_1q_2 + r$	$(1-q_1)q_2 - r$
k_2	$q_1(1-q_2) - r$	$(1-q_1)(1-q_2) + r$

Table 3.5.1. Interaction between I_i and k_i

$$\begin{aligned}
m(I_i, k_i) &= (q_1q_2 + r)(I_1 + k_1H)^\gamma + [(1-q_1)q_2 - r](I_2 + k_1H)^\gamma + \\
&+ [q_1(1-q_2) - r](I_1 + k_2H)^\gamma + [(1-q_1)(1-q_2) + r](I_2 + k_2H)^\gamma \\
&= q_1q_2(I_1 + k_1H)^\gamma + (1-q_1)q_2(I_2 + k_1H)^\gamma - \\
&+ q_1(1-q_2)(I_1 + k_2H)^\gamma + (1-q_1)(1-q_2)(I_2 + k_2H)^\gamma \\
&- r[((I_1 + k_2H)^\gamma - (I_1 + k_1H)^\gamma) - ((I_2 + k_2H)^\gamma - (I_2 + k_1H)^\gamma)]
\end{aligned}$$

Consider the expression inside r. Note that $(I_i + k_iH)^\gamma$ is equivalent to the health production function. Whether or not the covariance case would lead to higher community's health stock depends on the difference between the health level gained by the poor when health expsoure increases, and the gain of the rich when health exposure increases. If the poor gain more, then the covariance between income and exposure leads to higher health level.

Let us examine the cross derivative of $m(k_i, I_i)$ with respect to k_i and I_i

$$\frac{\partial m}{\partial k_i} = H\gamma(I_i + k_iH)^{\gamma-1} > 0$$

$$\frac{\partial^2 m}{\partial k_i \partial I_i} = H\gamma(\gamma - 1)(I_i + k_iH)^{\gamma-2} < 0$$

Therefore, if you have a high income, the gain in exposure is lower than when you have a low income.

$$\frac{\partial m}{\partial k_i}|_{I_i=I_1} > \frac{\partial m}{\partial k_i}|_{I_i=I_2}$$

When the health exposure k decreases, the poor's health level would increase more than that of the rich. Thus, their health level increases more than that of the rich when health exposure increases. As a result, $r[((I_1+k_2H)^\gamma-(I_1+k_1H)^\gamma)-((I_2+k_2H)^\gamma-(I_2-k_1H)^\gamma)] > 0$. Therefore, the covariance between income and exposure leads to lower level of health. It is because the gain of the poor from living in high healthy exposure environment is higher than that of the rich. This implies that the gain of the rich living in a high exposure environment is not enough to compensate the loss of the poor living in a less healthy exposure environment. In addition, the more segregated the community is, the less healthy the environment is, i.e. a high r leads to a low H^*.

As a result, the new equilibrium point H^{*2} will be smaller than in the case of intergration. Therefore, the poor has three disadvantages in a inequality and segregated community. First, a direct effect via a low income lessens his health level. Second, low interaction k_i with the community's health stock lowers his health. Third, these two direct effects affect the health stock, and lower the community health stock, which will come back and lower his health level.

3.6 Summary

This chapter examines how income inequality and residential segregation can affect an individual's health via decreasing the community's health.

1. An individual's health is an increasing and concave function with respect to the group income and the community's health. As a result, the community health has a unique and stable equilibrium.

2. Income inequality and variation of exposure independently decrease individual's health level via the decrease in the community's health

3. When variation of exposure happens to the extent that the poor lives close together, and the rich lives in a different neighborhood, then it is even worse than when the two exists separately. In other words, residential segregation decreases the community's health and, as a result, has a negative impact on an individual's health.

It is important to recognize that the conclusion comes from our assumption of concavity of health production function. Because the health production function is concave with respect to an individual's income income, redistribution of income and residence benefit the health of a community. In addition, our functional form is rather restrictive when comes to the relationship between I_i and h_i.

4
Conclusion

There are different ways that income inequality might affect an individual's health in general and infectious disease transmission in particular. This project builds a mathematical model that analyzes how income inequality and residential segregation can directly influence the community's health level and, as a result, decrease an individual's health. Chapter 1 presents an overview of the literature. Because the empirical studies report inconsistent results about the relationship between income inequality and residential segregation on mortality rates of various causes in developed countries, further investigation of this research question is needed in developed countries and even more so in the developing world. Furthermore, the literature does not provide a theoretical framework that establishes the casual links from inequality and segregation to health using a behavioral model. In Chapter 2 and Chapter 3, this project develops a formal model that explains how spatial concentration and income inequality might be an important aspect in public health intervention in countries where infectious diseases are highly prevalent. We assume that the health production function is concave with respect to income, and infectious diseases are subject to spatial externalities.

Chapter 2 presents a general model that explains the relationship between an individual's transmission with personal income (I_i), the exposure coefficient (k_i), income via the exposure coefficient (k_i is a function of I_i), and the community's health H. It analyzes the concavity of an individual's health production function with respect to each variable, and the concavity will later dictate how income inequality and segregation can affect disease transmission in Chapter 3. When one variable interacts with another, it can dampen or magnify the concavity of the individual's health production function with respect to that variable. In some cases such interaction can lead to a concavity change of the health production function. For instance, an increase in community's health leads to a decrease in medicine consumption if medicine is less effective in a healthy environment. This decrease in medicine consumption makes the individual's health production function less concave with respect to the community's health. If the decrease in medicine is, at some point, strong enough to outweigh the increase in the community health, the function might change to convex, indicating an increasing marginal return of an individual's health production function with respect to community health. On the other hand, a healthy environment and medicine can be complementary goods, i.e. medicine is more effective in a healthy environment. Independently, the community's health is concave with respect to an individual's health. Combining with the increase in medication consumption, the function of an individual's health with respect to community health might be convex. In addition, residential segregation might change the concavity of the health production function with respect to personal income. This change in concavity causes a double disadvantage to the poor: low income and low exposure to the healthy environment. Specifically, the low income group suffers from the lack of medicine and faces high exposure to the disease pool or low exposure to the health stock. These concavities of the individual's health production function dictates the environmental health response function's concavity ($s(H)$). This creates a possibility of multiple equilibria of the environmental health, causing a possible disease trap for a country.

Chapter 3 takes a closer look at how income inequality and segregation affect an individual's health via a specific functional form. They decrease the community health stock H^* and, as a result, decrease an individuals health h^{**}. Our functional form exhibits a diminishing marginal return with respect to each variable. After analyzing the intricate linkages between an individuals income, the groups income, medicine, and the communitys health environment, the final relationship between an individual's health and these variables exhibit diminishing marginal returns. The response community health function $(s(H))$ is also concave with respect to H. Therefore, there is only one unique equilibrium outcome of the community's health. Because we assume that both an individual's health and environmental production functions are concave with respect to personal income, group income, and the exposure coefficient, it is more beneficial to take away from the rich and give it to the poor as the gain of the poor exceeds the loss of the rich. Similarly, income segregation will lead to a low community health because the poor experiences both low income and low exposure. Overall, income inequality and segregation leads to a worse off outcome for the communitys health. After redistribution of income and space, the poor gain in three aspects: an increase in personal income (increase in I_1), a high exposure to the healthy environment (increase in k_1), and a higher level of community health (increase in H^*). The effect of redistribution on the rich is unclear. The rich experience decreases in income and exposure, but they enjoy a positive externality from an increased health stock. The overall impact of redistribution of income and residence on the rich depends on whether the community health effect is larger or the exposure and income effects dominate.

However, there are many limitations to the model. The functional form is restrictive and cannot demonstrate a possibility of multiple equilibria. Conclusions in Chapter 3 are derived from our essential assumption that the individual's health production function exhibits marginal diminishing returns with respect to income and exposure. Results will not be the same if this assumption does not hold. In addition, the model does not take into account other community characteristics formulated by inequality and segregation that might affect its member's health. Access to healthcare facilities, education level, and sanitary systems are examples of such neighborhood characteristics. Despite the limitations, we hope that the project illuminates the complex linkages between income inequality, residential segregation, and infectious diseases transmission. The model separates the direct of inequality and segregation with the indirect effect via community's quality. This separation, we believe, is important for different policy makers in public health intervention. The project discusses three possibilities to improve the health of a community. Firstly, the government can reduce income inequality by transferring income from the rich to the poor. As the gain of the poor exceeds the loss of the rich, the community's overall health increases. Second, the government can create an incentive for the rich to move to or not migrate out of poor neighborhoods and reduce residential segregation. As a result, the poor's exposure to the positive health stock increases, which help improve their health. The third way, which comes from the empirical discussion, is that the government can directly improve healthcare infrastructure in poor neighborhoods.

To that end, it is important to also recognize that the direct and indirect effect of inequality and segregation on health are related. Why is there an inequality? Why are people segregated? Why is it that areas that possess high income inequality and segregation usually experience a poor neighborhood quality? More often than not, inequality leads to segregation, and probably segregation causes underinvestment in a public space. Even if income inequality and segregation themselves do not directly decrease the health level of community or individual, it might still worth while to close the gap and integrate people. It is because the rich has an incentive in improving the neighborhood characteristics that the poor can benefit from such improvement.

In terms of empirical research, a multi-level study is the best approach to disentangle the interconnection between individual's characteristics, the neighborhood quality, income inequality, segregation, and other spatial factors. A multi-level study takes into account that an individual's characteristics such as age, gender, and education level influence his risk of getting an infectious disease. At the same time, this individual lives in a community, so the community's characteristics such as access to healthcare and a good sanitary system impact his health. It is difficult to explicitly study segregation in developing countries because these countries do not have a readily spatial data unlike in the US. A researcher has to collect his own household survey data and figure out a distance between one community to another to create a spatial variable. Lieberson's P index for isolation mentioned in the introduction is another proxy for segregation if we do not have a distance between two communities. A study using P index needs to have two levels of communities because, essentially, P index is the proportion of poor people in a region times the concentration of poor people in its sub-region. Choosing a community unit is also important because inequality and segregation might have an effect on cities and on the state level, but these impacts might become minimal when it comes to a more micro-level. Multi-level studies that investigate whether income inequality and residential segregation impact health and the community's level, such as villages or cities, are important for public health intervention in poor countries, especially when inequality and segregation are rising trends in the developing world.

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