Math 142: Worksheet 14

Use the computer program dfield to analyze the following differential equations.

1. Consider the following differential equation:

\[ \frac{dx}{dt} = xt \]

Use dfield to plot the direction field for this differential equation.

(a) If \( x(0) > 0 \), what do solutions to this differential equation look like?

(b) If \( x(0) < 0 \), what do solutions to this differential equation look like?

2. Consider the following differential equation:

\[ \frac{dx}{dt} = \frac{1}{10} (x^3 - 9x) \]

Use dfield to plot the direction field for this differential equation. Sketch some sample solutions to this differential equation.
3. Consider the following differential equation:

\[
\frac{dx}{dt} = x + t
\]

Use dfield to plot the direction field for this differential equation.

(a) If \(x(0) = 2\), what happens to \(x(t)\) as \(t \to \infty\)?

(b) If \(x(0) = -4\), what happens to \(x(t)\) as \(t \to \infty\)?

(c) If \(x(2) = -1\), what happens to \(x(t)\) as \(t \to \infty\)?

(d) If \(x(-3) = 0\), what happens to \(x(t)\) as \(t \to \infty\)?

(e) In general, for what initial conditions does \(x(t) \to \infty\), and for which does \(x(t) \to -\infty\)?
4. In last week’s homework, you used Euler’s method to analyze the following differential equation:
\[
\frac{dP}{dt} = P \left(1 - \frac{P}{5000}\right) - F
\]
where \( P \) was the population of fish in a lake and \( F \) was a constant equal to the rate at which fishermen remove fish from the lake.

(a) Use dfield to plot the direction field for this differential equation with \( F = 1000 \). Plot some of the solutions. What are the possible outcomes for the fish population? For which initial populations do these outcomes occur?

(b) Use dfield to plot the direction field for this differential equation with \( F = 2000 \). What are the possible outcomes for the fish population?

(c) Plot the differential equation for several different values of \( F \). Try to determine at what value of \( F \) the direction field changes from looking like the direction field in (a) to looking like the direction field in (b).