Math 142: Worksheet 12

1. Consider the following differential equation:

\[
\frac{dy}{dx} = y(y + 2)(y - 3)
\]

(a) What are the constant solutions to this differential equation? (That is, what are the solutions of the form \( y = \text{a constant} \)?)

(b) For what values of \( y \) is the solution increasing? For what values is it decreasing?

(c) If \( y(x) \) is a solution with \( y(0) = 2 \), what do you think \( \lim_{x \to \infty} y(x) \) is equal to?

(d) If \( y(x) \) is a solution with \( y(0) = -1 \), what do you think \( \lim_{x \to \infty} y(x) \) is equal to?

(e) If \( y(x) \) is a solution with \( y(0) = 5 \), what do you think \( \lim_{x \to \infty} y(x) \) is equal to?
2. Consider the following differential equation:

\[ \frac{dy}{dx} = y^2 + A \]

where \( A \) is a constant.

(a) If \( A < 0 \), what are the constant solutions? If \( A > 0 \), what are the constant solutions?

(b) Suppose that \( A < 0 \). For what values of \( y \) is the solution increasing? For what values of \( y \) is the solution decreasing?

(c) Suppose that \( A > 0 \). For what values of \( y \) is the solution increasing? For what values of \( y \) is the solution increasing?

(d) If \( A = -2 \) and \( y(0) = 1 \), what do you think \( \lim_{x \to \infty} y(x) \) is equal to?

(e) If \( A = -2 \) and \( y(0) = -3 \), what do you think \( \lim_{x \to \infty} y(x) \) is equal to?