1. An object is placed in a refrigerator. The temperature $T$ of the object (measured in Fahrenheit) satisfies the following differential equation:

$$\frac{dT}{dt} = k(M - T)$$

where $M$ is the temperature of the refrigerator, $k$ is a constant that depends on the object, and $t$ is measured in minutes.

(a) Find all solutions to the differential equation.

$$\int \frac{dT}{M - T} = \int k \, dt$$

$$- \ln |M - T| = kt + c$$

$$\ln |M - T| = -kt + c$$

$$|M - T| = e^{-kt + c}$$

$$M - T = \pm e^{-kt} e^c$$

$$M - T = Ae^{-kt}$$

$$T = M - Ae^{-kt}$$

(b) If $M = 38$, $k = 0.02$, and $T(0) = 70$, determine the temperature of the object after 30 minutes.

$$T = 38 - Ae^{-0.2t}$$

$T(0)$ = 70:

$$70 = 38 - Ae^0$$

$$70 = 38 - A$$

$$\Rightarrow A = -32$$

$$T(30) = 38 + 32e^{-0.2(30)} = 55.5620^\circ F$$
2. The population of fish in a lake satisfies the differential equation

\[ \frac{dP}{dt} = P \left( 1 - \frac{P}{5000} \right) \]

where \( P \) is the number of fish in the lake and \( t \) is measured in years.

(a) What are the constant solutions (solutions of the form \( P = \) a constant)?

\[ 0 = P \left( 1 - \frac{P}{5000} \right) \]

\[ P = 0, \ P = 5000 \]

(b) For what values of \( P \) is the solution increasing? For what values of \( P \) is the solution decreasing?

\[ 0 < P < 5000 \text{ increasing} \]

\[ P < 0, \ P > 5000 \text{ decreasing} \]

(c) Suppose that there are initially 2000 fish in the lake. What is \( \lim_{t \to \infty} P(t) \)?

\[ P = 5000 \]

\[ \lim_{t \to \infty} P(t) = 5000 \]

(d) Suppose that there are initially 7000 fish in the lake. What is \( \lim_{t \to \infty} P(t) \)?

\[ \lim_{t \to \infty} P(t) = 5000 \]
3. Now, suppose that fishermen remove fish from the lake at a rate of $F$ fish per year, so that the population of fish in the lake satisfies the following differential equation:

$$\frac{dP}{dt} = P \left( 1 - \frac{P}{5000} \right) - F$$

(a) Suppose that $F = 1000$ and $P(0) = 7000$.

i. Use Excel and Euler’s Method with stepsize $h = 0.05$ to estimate $P(5)$.

$$P(5) \approx 3764.5585$$

ii. Use Excel to graph an approximate solution. Print the graph, and attach it to your homework.

iii. What eventually happens to the fish population?

"It goes to approximately 3618 fish."

(b) Suppose that $F = 2000$ and $P(0) = 7000$.

i. Use Excel and Euler’s Method with stepsize $h = 0.05$ to estimate $P(5)$.

$$P(5) \approx 569.8579$$

ii. Print the graph, and attach it to your homework.

iii. What eventually happens to the fish population?

"The fish population dies out."