1. An object is placed in a refrigerator. The temperature $T$ of the object (measured in Fahrenheit) satisfies the following differential equation:

$$\frac{dT}{dt} = k(M - T)$$

where $M$ is the temperature of the refrigerator, $k$ is a constant that depends on the object, and $t$ is measured in minutes.

(a) Find all solutions to the differential equation.

(b) If $M = 38$, $k = 0.02$, and $T(0) = 70$, determine the temperature of the object after 30 minutes.
2. The population of fish in a lake satisfies the differential equation

\[
\frac{dP}{dt} = P \left( 1 - \frac{P}{5000} \right)
\]

where \( P \) is the number of fish in the lake and \( t \) is measured in years.

(a) What are the constant solutions (solutions of the form \( P = \text{a constant} \))?

(b) For what values of \( P \) is the solution increasing? For what values of \( P \) is the solution decreasing?

(c) Suppose that there are initially 2000 fish in the lake. What is \( \lim_{t \to \infty} P(t) \)?

(d) Suppose that there are initially 7000 fish in the lake. What is \( \lim_{t \to \infty} P(t) \)?
3. Now, suppose that fishermen remove fish from the lake at a rate of $F$ fish per year, so that the population of fish in the lake satisfies the following differential equation:

$$\frac{dP}{dt} = P \left( 1 - \frac{P}{5000} \right) - F$$

(a) Suppose that $F = 1000$ and $P(0) = 7000$.

i. Use Excel and Euler’s Method with stepsize $h = 0.05$ to estimate $P(5)$.

ii. Use Excel to graph an approximate solution. Print the graph, and attach it to your homework.

iii. What eventually happens to the fish population?

(b) Suppose that $F = 2000$ and $P(0) = 7000$.

i. Use Excel and Euler’s Method with stepsize $h = 0.05$ to estimate $P(5)$.

ii. Print the graph, and attach it to your homework.

iii. What eventually happens to the fish population?