1. In this problem we will approximate the value of $\int_{0}^{1} \sin(\pi x^2) \, dx$.

(a) The following picture shows nine rectangles that approximate the area under the curve $f(x) = \sin(\pi x^2)$. Each rectangle has width 0.1.

Make a table showing the height and area of each rectangle. Then, determine the total area of the nine rectangles.
(b) The following picture shows the area approximated by trapezoids and triangles, each with width 0.1.

Make a table showing the height and area of each trapezoid or triangle. (Note: the area of a vertical trapezoid is the width multiplied by the average height). Then, determine the total area of the trapezoids and triangles.

(c) Use your calculator to determine the actual value of the integral.
2. Suppose that \( f(x) \) is an increasing function on the interval \( 0 \leq x \leq 1 \).

(a) If we approximate \( \int_0^1 f(x) \, dx \) using rectangles with right endpoints, is the approximation an underestimate or an overestimate? Explain your answer by drawing a picture.

(b) If we approximate \( \int_0^1 f(x) \, dx \) using rectangles with left endpoints, is the approximation an underestimate or an overestimate? Explain your answer by drawing a picture.
(c) Suppose \( f(x) \) is the following function:

If we approximate \( \int_{0}^{1} f(x) \, dx \) using trapezoids, is the approximation an underestimate or an overestimate? Explain your answer.

(d) Suppose \( f(x) \) is the following function:

If we approximate \( \int_{0}^{1} f(x) \, dx \) using trapezoids, is the approximation an underestimate or an overestimate? Explain your answer.
3. When we use rectangles or trapezoids to estimate the value of an integral, we are approximating the given function with linear functions. In this problem we will approximate a function with a quadratic function instead of a linear function, and then use the quadratic function to estimate the value of an integral.

(a) Consider the function $f(x) = e^{x^2}$ for $-1 \leq x \leq 1$:

Determine the values of $f(0)$, $f'(0)$, and $f''(0)$. 
(b) Find values for $a$, $b$, and $c$, so that the parabola $y = ax^2 + bx + c$ has the same value, derivative, and second derivative at $x = 0$ as the function $f(x)$.

(c) Estimate the value of $\int_{-1}^{1} e^{x^2} \, dx$ by computing $\int_{-1}^{1} (ax^2 + bx + c) \, dx$.

(d) Use your calculator to determine the actual value of $\int_{-1}^{1} e^{x^2} \, dx$. 

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