1. Suppose that the population of a country is modeled by the following differential equation:

\[ \frac{dP}{dt} = kP - m \]

where \( k \) and \( m \) are both constants, with \( k > 0 \) and with \( m \) equal to the emigration rate.

(a) Find the solution to the differential equation that satisfies the initial condition \( P(0) = P_0 \).
(Your answer will involve the constants \( m \), \( k \), and \( P_0 \).)

(b) What condition on \( m \) (in terms of \( k \) and \( P_0 \)) will lead to exponential expansion of the population?

(c) What condition on \( m \) (in terms of \( k \) and \( P_0 \)) will result in a constant population?

(d) What condition on \( m \) (in terms of \( k \) and \( P_0 \)) will result in a population decline?
In this problem, you will use Excel and a variation of Euler’s method to analyze a second-order differential equation that models the motion of a block.

Consider the following mass-spring system:

![Diagram of a mass-spring system](image)

In this system, a block is attached to a wall by a spring. If we assume that the surface is frictionless, then the position of the block can be modeled by the following differential equation:

\[ mx'' = kx \]

where \( m \) is the mass of the block, \( x \) is the distance of the block from its equilibrium position at time \( t \), and \( k \) is a constant that depends on the spring.

(a) Suppose that \( m = 1 \), \( k = -4 \), \( x(0) = 2 \), and \( x'(0) = -1 \). What is \( x''(0) \)?

(b) Use \( x(0) \) and \( x'(0) \) to estimate \( x(0.02) \). (Use a linear approximation; that is, use Euler’s method).

(c) Use \( x'(0) \) and \( x''(0) \) to estimate \( x'(0.02) \). (Use a linear approximation of \( x'(t) \)).

(d) Use your estimate of \( x(0.02) \) from (b) to approximate \( x''(0.02) \).
(e) Fill in the following chart by continuing to approximate the solution \( x(t) \) using stepsize \( h = 0.02 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( x' )</th>
<th>( x'' )</th>
</tr>
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<tr>
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<td>2</td>
<td>-1</td>
<td></td>
</tr>
<tr>
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</table>

(f) Use Excel to continue Euler’s method with stepsize \( h = 0.02 \) for \( t \) between 0 and 10. Graph the resulting solution. Print the graph and attach it to your homework.

(g) Describe the motion of the block.

(h) We have been assuming that the surface is frictionless. If there is friction, then the motion of the block can be modeled by the following differential equation:

\[
mx'' = kx - px'
\]

where \( p \) is a constant that depends on the surface.

i. Suppose that \( m = 1, k = -4, p = 1, x(0) = 2, \) and \( x'(0) = -1 \). Use Excel to graph an approximate solution. Print the graph and attach it to your homework.

ii. Describe the motion of the block.