Statement of Teaching and Research Goals

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1 Teaching

Since arriving at Bard, I have enjoyed the freedom I am afforded in my teaching. Not only may I teach my classes how I want, with complete control over my syllabus, but I have also been able to develop my own new courses and influence the curriculum of the Mathematics Program. I am also quite pleased with the level of the Bard students I have taught, and once I came to understand how best to motivate them, I have been extremely impressed by their intensity and focus. This is particularly apparent in the student research I have advised, giving the students the opportunity to synthesize their knowledge and contribute something new to the field, and these projects are the most satisfying aspect of my teaching.

1.1 Teaching Style

In my classroom, I maintain an informal environment, where the students feel comfortable and are encouraged to ask questions, discuss their ideas, and participate in the class. With the small class sizes at Bard, I am able to tailor my teaching to the students, both adjusting the level to match that of the class, and also addressing the topics that most interest them. I try to avoid over-preparing my overall syllabus and individual classes. Instead, I approach a course asking myself what are the fundamental concepts of the subject that I most want the students to understand and retain, and I plan each class around the two or three ideas that I want them to learn that day. This affords me the flexibility to adjust to the needs of the students. In fact, some of my best classes have been unplanned digressions, where a student asks an interesting question, and letting the students guide the discussion, we head in an unexpected direction.

In my first semester at Bard, my teaching was more rigid, as I had come from a university with larger class sizes and a more structured curriculum. However, by my second semester, I started to understand the strengths, interests, and needs of Bard students. When I taught Topology in Spring 2008, I had a very advanced class, with most of the students heading to graduate school. At their request, I adjusted my plans to teach a standard course on Point-Set Topology, instead devoting the last month to an introduction to graduate level Algebraic Topology. On the other hand, when I taught Linear Algebra in Spring 2009, I adjusted my syllabus to include more time spent on applications, leaving less time for the abstract theory. Nevertheless, I made sure that they mastered what I view as the two most important theoretical topics: appreciating the ubiquity of abstract vector spaces in pure and applied mathematics, and the ability to compute the matrix for a linear transformation given a basis. For the handful of more advanced students in my Linear Algebra class, I provided them with extra theoretical readings, to keep them interested and better prepare them for further work in the subject.

Whenever I can, I show my students how the material I am presenting is relevant to my own work. For instance, whenever I attend a conference, I explain to the class the work that I presented there or some interesting ideas that I saw. I have explained the supersymmetry that I study to my Calculus class in terms of trying to take a square root of a derivative, and to my Linear Algebra class in terms of finding matrices that satisfy certain conditions. I have even given my String Theory and Advanced Linear Algebra classes problems from my own research. In Linear Algebra, I demonstrated the animation built into iPhone apps as an example of affine transformations, showing that the developer documentation describes affine transformations just like we discussed in class. In Data Structures, I often contrasted Java constructions with analogous constructions in Objective-C and other languages that I have used in my own programming. This shows the students that I am invested in the mathematics that I teach, while also piquing their curiosity about mathematical research or professional computer programming. Three of the six research projects I have advised have been on topics related to my research: Clifford algebras, codes, and Adinkra diagrams. In Spring 2008, I arranged for Zach Hamaker to present his senior project work to several members of my research group, and in Spring 2009, I arranged a meeting with my student Sylvia Naples met and my collaborator Jim Gates, to show him her extraordinary work which we will soon turn into a joint paper.

I try to be as friendly and approachable as possible, but I am surprised that more students do not come to my office hours. Those who do come tend to benefit from the on-on-one interaction and get a lot more out of the course. When I was at the University of Oregon, students regularly packed my office asking questions, but at Bard I usually see only a small handful of students. I plan to take action to address this. In Spring 2009, I suggested to several students on their midterm crite sheets that they should come talk to me, and several of them did. In the future, I may schedule office hours in the early evening when more students are free, or hold them in a classroom as an unofficial help session. I may also require my students to meet with me at least once before midterms to help them get over any initial shyness. The main way I have found to drive students to my office hours in my more advanced classes is to give them independent projects to present at the end of the semester. The first time I did this, in my Fall 2007 Linear Algebra course, many of the students threw something together at the last minute. Learning from this, I now schedule into my syllabus several milestones, requiring my students to turn in a proposal, outline, rough draft, and final draft, consulting with me at each stage. This guarantees that the students start thinking about their projects sooner, but most importantly it gets them talking to me.

1.1.1 Problem-Oriented Teaching

I spend as much as half of my class time going over problems, whether they are prepared examples of the idea I just presented, or homework problems that the students ask about. Mathematics requires participation; it is best learned by doing it yourself, rather than watching a professor work a problem on the blackboard. To this end, I assign problems that go beyond the techniques I discuss in class, letting the students read ahead in the textbook and explore more advanced ideas, the theoretical background, and unconventional applications of the material. Some students find this disconcerting, as they have grown accustomed in their high school mathematics classes to simply repeating on the homework the same computations that were done in class. Instead, by forcing them to explore ideas first on their own, I let their homework questions drive the class; students learn and understand the answers better when they ask the questions. The difficult homework also drives students to my office hours, where they benefit from one-on-one discussions.

I believe that this problem-oriented style of teaching applies equally well to classes involving computer programming. In Fall 2008, I taught Data Structures in the Computer Science program. This was the first time I had taught a purely computer science course. My students found the class to be a lot of work, but I believe they learned all the more for their effort. I gave them labs that tested their understanding of the concepts, but also pushed them into unfamiliar territory. In class, I emphasized the underlying computer science topics, while saving the Java syntax and programming issues for the labs. Rather than teaching them all the methods and techniques, I showed them where to find the appropriate online documentation and sample code. On the other hand, I did not leave them completely to their own devices: the labs gave me the opportunity to work with the students individually, and each week I did an in-class code review, taking one student's lab work and having the whole class evaluate its correctness, efficiency, and style. This approach came from my experience programming in the real world, where your knowledge of a particular language is less important than your ability to adapt to new languages and frameworks. Indeed, this reflects my view that the role of a teacher is not to provide information, but rather to give the students the tools they need to seek out and understand that information themselves.

1.2 Curricular Development

In the last few years the Mathematics Program has increased dramatically, in terms of the number of majors, the size of the faculty, and the number of courses we teach. This has required the program to expand its course offerings in several ways. Here I describe the courses that I have developed and some of the curricular changes I have made to better suit the needs of our students. In the future, as the Mathematics Program stabilizes at its current size or perhaps even continues to grow, we need to develop a long term plan for our course offerings. Up to now, our course planning has been ad hoc, adjusting each semester to the anticipated demands of our majors, and this flexibility has worked extremely well for us. A multi-year plan, with various advanced and applied courses in a two or three year rotation, will be particularly helpful at moderation, where we look ahead to the next two years of study. I hope that some of the courses I discuss below will become part of this rotation.

With all of the sections of Calculus I and II that we now offer, as well as the multiple sections of some higher level courses, I think the Mathematics Program should attempt to standardize the material covered in these courses. Rather than proposing a uniform syllabus, which would diminish the freedom our faculty enjoys, I propose appointing course coordinators for these multi-section classes. I benefited Spring 2008 by regularly discussing our Calculus I classes with John Cullinan, and I have seen other faculty comparing notes at our Math Table lunches. Having a course coordinator and course meetings would formalize this arrangement, making sure the same basic topics are covered, and helping advise new or visiting faculty. At the University of Oregon, I was such a course coordinator for Linear Algebra, and I believe I can bring this experience to Bard.

1.2.1 Linear Algebra

With the Bard senior project, we see how well our students really understand and retain the material from their classes. Rather than just looking at their test scores and grades from the end of the semester, we see whether the students can apply their knowledge in their independent research. One thing that struck me when I arrived at Bard was that despite their mathematical sophistication, our students were quite weak in linear algebra.

All but one of the six projects I have advised (including a BPI senior project and an MAT academic research project) have involved linear algebra, which is a reflection not only of my mathematical interests, but also of the fundamental role that linear algebra plays in advanced theoretical and applied mathematics. In all of these cases, I needed to review the fundamental concepts of linear algebra with them before proceeding.

I taught a tutorial in Spring 2008 on Applied Linear Algebra to Dexin Zhou, who went on to do a senior project with me. This allowed me to determine what he knew and fill the gaps in his background. Once I had gauged our students' understanding of linear algebra, I developed a 400-level Advanced Linear Algebra class in Fall 2008. Such a course is vital to prepare our students for graduate school, where a strong background in theoretical linear algebra is often assumed but not taught. The goal of this course was to teach the linear algebraic techniques likely to arise in research and future courses, including multi-linear algebra and module theory. For my final exam, I gave problems that had come up in my own research, reasoning that if my students could solve problems which had appeared "in the wild", then they were well prepared for any linear algebra they were likely to encounter.

I taught the 200-level linear algebra course at Bard twice, Fall 2007 and Spring 2009. I used some of the same techniques both times, such as the Online Row Reducer and Linear Algebrator interactive software tools I wrote to teach the Gauss-Jordan row reduction and Gram-Schmidt orthogonalization algorithms. However, in many other respects my approach changed between the classes. In Fall 2007, I gave all take-home exams, but in Spring 2009, I gave all in-class exams. This change has two effects: first it prepares them for timed exams such as the GREs, on which Bard students often do not perform well; and second, it forces them to study hard and review all the material, rather than just what they need for the few particular exam problems. In my approach to applications, in Fall 2007 I presented a long list of individual applications. In contrast, in Spring 2009, I emphasized the common features and types of these applications, better training the students to recognize linear algebra problems wherever they appear. Finally, I changed my approach to the theory. In Fall 2007, many of the students were more advanced, having already taken Proofs and Fundamentals, so I taught a more theoretical course which the most advanced students still found too elementary. My Spring 2009 course had less advanced students, including a few first years, so for the theoretical aspects I emphasized the need to use the definitions precisely, and I took them step-by-step through a few simple proofs.

1.2.2 Numerical Analysis and Applied Mathematics

Another pressing need of the Mathematics Program is for applied courses. The program has been searching for several years now for an applied mathematician, or for someone who can teach applied courses and advise applied student research. Although I am very much a theoretical mathematician, my research extends into physics, and I also have experience as a professional computer programmer. Mathematical software is growing in importance, and mathematics is becoming more "experimental". The ability to use software for numerical or symbolic computation gives students a distinct advantage, allowing them to work examples and explore possibilities beyond what they could do with pencil and paper. I am particularly interested in the mathematical software Sage, a Python-based computation engine that has been extraordinarily useful in my research.

While at the University of Oregon, I was charged with redesigning the Numerical Analysis sequence, and I took it from one of the least liked courses in the department to one with a perfect student evaluation. I brought this experience to Bard, teaching a 2 credit Numerical Analysis Lab using Sage. I had to distill the material from a full year down to a half-course, so I focused on Taylor series, root approximation, linear algebra, and curve fitting. Since most of the textbooks on the subject are quite dry and present either cookie-cutter numerical recipes or overly complicated proofs, I wrote my own notes tailored to the 2 credit format, focusing on one idea each week and accompanying it with a programming problem. This proved to be a great deal of work, between writing my notes and the solutions to the problems, and helping my students debug their programs. However, I am quite pleased with the outcome, and I have included these course materials with my tenure file. I am looking forward to teaching this course again, but for a full 4 credits, allowing me go beyond basic Sage programming to the surprisingly interesting underlying theoretical ideas.

I used my numerical analysis expertise to teach a tutorial on numerical differential equations to Peter Golbus in Spring 2008. I plan to reprise some of this material in my Differential Equations class in Fall 2009, spending a few weeks exploring numerical solutions to complement the theoretical solutions discussed in the rest of the class. I hope to teach Calculus II in the near future, where I plan to introduce basic numerical calculus and in particular the numerical applications of Taylor series. My other applied classes include a Fall 2008 physics tutorial with Marjorie Schillo on Quantum Field Theory, and an upcoming Fall 2009 course on Coding Theory, both of which are related to aspects of my supersymmetry research. In addition, I have advised two applied senior projects: Dexin Zhou on financial mathematics, and Zhechao Zhou on a joint physics-mathematics project on network connectivity.

1.2.3 Problem Solving

In the Falls of 2007 and 2008 I offered a two credit Problem Solving Seminar. One purpose of these classes was to prepare students for the Putnam Mathematical Competition, given each year to students throughout the USA and Canada on the first Saturday of December. This is an extraordinarily difficult exam, with many students unable to score even a single point. This class was not about teaching the students new material, but rather techniques to solve hard problems using the mathematics they already knew. My goal in this class was to push students outside their comfort zones, so they would no longer be intimidated by difficult problems or unfamiliar topics, instead attacking them repeatedly from a variety of different directions, To balance this otherwise stressful and occasionally competitive nature of the class, I attempted to make the class as physically comfortable as possible, offering it over dinner at Kline one night a week and grading it pass/fail only. As a direct result of this class, participation in the Putnam has increased from around ten students in 2007 to two dozen in 2008. In addition, the Bard team in 2008 placed 41st out of 545 institutions, and a record 14 of our students managed to score at least one point!

1.2.4 String Theory

When I interviewed for my position at Bard, one of the ideas I proposed was an introductory course on the mathematics of string theory for non-majors. I developed this course and taught it in Spring 2009 to an assortment of students with a minimum Calculus I background. My purpose was to use the students' interest in string theory to introduce them to various advanced topics in mathematics and physics, such as complex numbers, quaternions, differential equations, topology, differential geometry, and supersymmetry. Although a conventional course on string theory requires graduate study, many of the topics can be appreciated qualitatively. For example, it is not necessary to take courses in Calculus III, Differential Geometry, and General Relativity to understand how a string moving in a curved space inherits its own curvature, and thereby exhibits its own gravity. On the other hand, because my students knew calculus, I was able to fill in many of the details not given in the popular accounts of string theory, such as showing them the explicit equations for Feynman path integrals, and how the Dirac equation predicts the presence of antimatter.

I was particularly pleased to be able to share my supersymmetry research with my string theory students. Supersymmetry is typically an extremely advanced topic, but the Adinkra diagrams that my collaborators and I study give a significantly simpler yet no less rigorous way of describing supersymmetry in terms of pictures. (Jim Gates has even used these Adinkras to do publishable supersymmetry research with high school students.) My final exam took students to the cutting edge of mathematical physics research, working out a simple example of a formula that I had discovered only the week before!

In addition to giving my students a glimpse of advanced research and an appreciation of the necessity of funding particle accelerators, my aim was to show them how mathematicians think. Mathematical thought is different from the critical thinking they learn elsewhere in the college, even in the sciences. Thinking extraordinarily literally and logically, mathematicians require a precise definition for every term, and every idea, no matter how simple, must be completely and undeniably justified. This is a valuable skill to learn, and it can be applied wherever mathematics appears in everyday life, from understanding the collapse of the financial markets to the risks associated with swine flu.

2 Research Overview for Non-Specialists

Note. This overview of my research parallels my Spring 2009 Faculty Seminar on *Symmetry and Supersymmetry*. I am including the slides from this seminar with my tenure file as they contain colorful illustrations of several of the ideas mentioned here.

My recent research is divided into two separate but related programs: K-theory in equivariant symplectic geometry [25–27], and off-shell supersymmetry in physics [7–17]. The common thread between the two is the study of symmetry. A symmetry is a way of transforming an object which leaves some of its fundamental properties unchanged. This can be as simple as the left-right reflective symmetry of a human face, or more complicated like the many rotational symmetries of a soccer ball. Most of the symmetries that I study are called Lie (pronounced Lee) symmetries. Examples include the rotations of a circle, or the rotational transformations of spacetime used in special relativity. Such symmetries can often be expressed in terms of angles, which vary continuously over some range, usually the interval from 0 to 2π introduced in trigonometry.

2.1 Equivariant symplectic geometry and K-theory

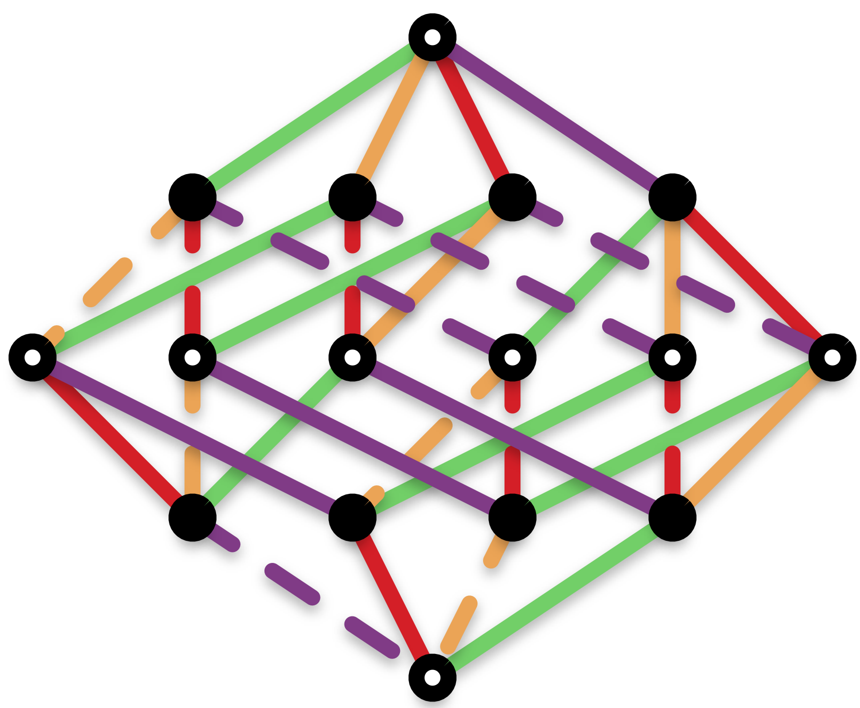
In physics, symmetries come with corresponding conserved quantities. For example, the fact that energy and momentum can neither be created nor destroyed is a consequence of the symmetry of the laws of physics under shifts in time and space, respectively. These physical symmetries are fundamental to the scientific method, as they say that results must be reproducible at a later time or different location. *Equivariant symplectic geometry* is the mathematical abstraction of this physical concept, studying curved spaces with the property that every symmetry gives rise to a conserved momentum. The standard example is the surface of the earth, with the symmetry given by the earth's rotation about its axis through the North and South Poles; the corresponding conserved momentum is the latitude, which does not change as the earth rotates. By studying both the symmetries and their momenta, one can learn a great deal about the curvature of the space.

The tool I use to study curved symplectic spaces is called *K-theory*, which measures the ways these spaces can be twisted. The standard example of a twist is a Möbius strip, which can be constructed out of a long thin strip of paper, by joining the two ends together with a half-twist. If we, like M. C. Escher, imagine an ant crawling around this strip, when it returns to its original position it is surprised to find that it is on the opposite side of the paper. K-theory measures such twists from the point of view of the ant, asking how the ant's world could possibly change when it walks a complete loop. In particular, if we were building a Möbius strip but instead of a half-twist we used a full-twist, the ant would not notice the twisting, as it would return to its starting point on the same side of the paper; such full-twists are not measured by K-theory, as they are visible only from the extrinsic point of view of an external observer, not intrinsically from the point of view of the ant. For spaces more complicated than a circle, it is usually not possible to imagine the extrinsic twists, as they require more than three dimensions. However, we can still appreciate the intrinsic twists. For instance, if we were to twist the surface of the earth, we could ask how our world might have changed after we walk around the equator. It is hard to visualize such twists, but suppose we held a ribbon in each hand, one attached to the north pole and the other to the south pole. Then we could measure the possible twists of the surface of the earth by seeing how many times each ribbon is twisted when we return to our starting point.

2.2 Off-shell supersymmetry in physics

In mathematics, a *supersymmetry* is a transformation between two distinct yet equal objects, which leaves their combined properties unchanged. My particular interest is in supersymmetry in physics, where the objects in question are the two fundamental classes of elementary particles: bosons, which transmit force and energy, and fermions, which constitute matter. Although these two types of particles appear and behave quite differently, supersymmetric physics posits that the laws of physics remain the same if we exchange the roles of pairs of bosons and fermions, called superpartners. In fact, supersymmetry tells us that there is not just one superpartner pairing, but in fact several; just as the laws of physics are unchanged by moving in the three separate x, y, and z directions, the superpartner exchanges correspond to motions in new "odd" directions, described mathematically using quantum mechanics.

Multiple superpartner pairings intertwine the bosons and fermions into a mesh of particles, called a supermultiplet. In 2004, my research collaborators Mike Faux and Jim Gates introduced diagrams called *Adinkras* (named after symbols in African art) that show this supersymmetric structure graphically, condensing the otherwise complicated equations into significantly simpler, yet no less rigorous, pictures. In fact, these diagrams are simple enough that I have used them quite successfully to explain supersymmetry in my non-majors String Theory class, and Jim Gates has even used them to do research with high school students.



Above is the hypercube Adinkra with open and closed vertices corresponding to bosons and fermions, respectively. It describes superpartner pairings for four odd directions, given by following the green, yellow, red, and purple edges. The heights of the vertices and the dashed edges give the additional information necessary to reconstruct the supersymmetric equations. It is drawn using the *Adinkramat*, a program I wrote for Adinkra drawing and computation.

My supersymmetry research group consists of the physicists Jim Gates, Tristan Hübsch, and Mike Faux, as well as the mathematicians Chuck Doran and Kevin Iga. Our goal is to describe all possible supermultiplets, i.e., classify all the ways that bosons and fermions could possibly assemble together under the rules of supersymmetry. Such complete classification problems are quite common in mathematics, although physicists tend to be interested in only a subset of the possibilities satisfying various additional physical conditions. Of particular interest to the physicist are the supermultiplets corresponding to 4, 10, and 11 spacetime dimensions, with 16 or 32 odd directions worth of superpartner pairings, incorporating particles of matter, quantum mechanical forces, and gravitation. These supermultipets have 16 or 32 different superpartner pairings, and a minimum of 256 or 65536 particles, respectively. We are studying these large supermultiplets both computationally, with the aid of a supercomputer, and theoretically, combining physical methods with graph theory and algebra. Our hope is that our techniques will prove useful in describing the supersymmetry required by string theory, or that supersymmetry fitting our descriptions will be found in the near future at the Large Hadron Collider (LHC) in Switzerland.

My particular interest in this research program is in the theoretical underpinnings of the Adinkra diagrams and supermultiplets. I classified how these possible diagrams can be obtained as quotients of hypercubes, which led us to discover a surprising connection between supersymmetry and error correcting codes (such as are used to safely transmit information over the internet or encode it onto CDs). I also tied our supersymmetry research to the algebraic problem of studying filtered Clifford supermodules, translating our physics problem into one of pure mathematics. Working with my research group and my senior project student Sylvia Naples, I closely examined the possible assignments of units to the particles in a supermultiplet, viewed in terms of the heights of the vertices on the Adinkra, providing many examples and counterexamples useful in the classification of Adinkras. In addition, my senior project student Zach Hamaker and MAT academic research project student Anna Casteen worked on algebraic and coding theory problems arising in my research, respectively. I also worked with my future senior project students Dexin Zhou and Zhechao Zhou over the summer of 2008, writing software in Sage for manipulating Adinkras. Since arriving at Bard, the focus of my advanced research has been on reconciling the structure of Adinkras and their supermultiplets with the rotational symmetries of spacetime required by special relativity; these results will be described in two forthcoming papers.

My supersymmetry research also provides opportunities for collaboration with other Bard mathematics faculty. Sam Hsiao has suggested that my Adinkra diagrams can be viewed as posets, his area of expertise, with the heights of the vertices determining a partial ordering on the bosons and fermions. From our preliminary discussion, it appears that the language and theory of posets may be able to contribute to my study of Adinkras. John Cullinan has pointed out that the error correcting codes that arise in the classification of Adinkras also have connections to the study of finite simple groups of symmetries. In addition, Jim Belk has expressed an interest in both the discrete and algebraic structure of Adinkras. In my discussion of service in Section 4.1 below, I describe my plans to organize a Mathematics Faculty Research Seminar with the goal of promoting such collaborations among our faculty.

3 Research Details for Specialists

I am currently pursuing two research programs: First, I work in equivariant symplectic geometry and topology, establishing techniques for computing the K-theory of Hamiltonian G-spaces and their symplectic quotients. Second, I collaborate with several mathematicians and physicists to classify the representations of off-shell supersymmetry arising in field theory, using techniques involving graph theory, coding theory, and algebra. Both projects grew out of my earlier research [35–38] which used Dirac operators to probe the representation theory of Lie groups and Lie superalgebras in terms of twisted and equivariant K-theory.

3.1 Equivariant symplectic geometry and topology

A *symplectic manifold* (M, ω) is a manifold M equipped with a closed, non-degenerate 2-form $\omega \in \Omega^2(M)$. In particular, all 2-dimensional surfaces and Kähler manifolds are automatically symplectic, and symplectic geometry plays a significant role in the study of 4-manifolds. If G is a Lie group, then a *Hamiltonian G-space* is a symplectic manifold (M, ω) on which G acts preserving the symplectic form ω, which further admits a G-equivariant *moment map* $\mu : M \to \mathfrak{g}^*$ satisfying Hamilton's equation $\langle d\mu, Z \rangle = \iota_{Z^\sharp}\omega$ for all $X \in \mathfrak{g}$, where $Z^\sharp$ denotes the vector field on M generated by the infinitesimal action of $Z \in \mathfrak{g}$. Equivariant symplectic geometry studies the geometry and topology of such Hamiltonian spaces, as well as the symplectic manifolds constructed from them.

If (M, ω, μ) is a Hamiltonian G-space and 0 is a regular value of μ, then G acts locally freely (i.e., with finite stabilizers) on $\mu^{-1}(0)$, and the *symplectic quotient* of M is

$$M /\!/ G := \mu^{-1}(0)/G,$$

viewed as the standard quotient if the action is free, or an orbifold otherwise. Symplectic quotients are themselves symplectic, and this process allows us to create new and interesting examples out of simpler equivariant ones. This construction originated in mathematical physics, where moduli spaces of fields satisfying a differential constraint modulo gauge invariance can be treated as symplectic quotients where the differential equations are interpreted as the condition $\mu = 0$ for the moment map of the gauge action (see, for example [3]). Symplectic quotients are related to Geometric Invariant Theory ("GIT") quotients of Kähler manifolds by complex group actions, which give symplectic realizations of moduli spaces in algebraic geometry. Other examples of symplectic quotients include toric varieties, and there are links between the topology of symplectic quotients and representation theory.

The goal of this research program is to compute topological invariants of symplectic manifolds in terms of (equivariant) cohomology theories. The bulk of the literature in equivariant symplectic geometry works with rational or de Rham cohomology. (See, for example, [22, 24, 33].) One possible refinement is to work with integral cohomology, but in the presence of torsion, many of the results and computations in equivariant symplectic geometry become significantly more complicated (see, for example, [31, 40]).

My work, in collaboration with Megumi Harada at McMaster University and Reyer Sjamaar at Cornell University, takes an alternative approach, studying torsion not through integral cohomology, but rather via complex K-theory. A generalized cohomology theory built from isomorphism classes of complex vector bundles (see [2, 29]), K-theory contains all the information of rational cohomology, but it is also an integral theory with slightly different (often more interesting) torsion than integral cohomology. In our work [25, 27], we have shown repeatedly that the torsion difficulties in integral cohomology disappear in K-theory.

3.1.1 The K-theory of symplectic quotients

One of our aims is to compute the K-theory of symplectic quotients. In [33], Kirwan studied their rational cohomology by means of the map

$$\kappa : H_G^*(M) \to H_G\big(\mu^{-1}(0)\big) \cong H^*(M /\!/ G),$$

showing that it is surjective using Morse theory for the square of the norm $\|\mu(x)\|^2$ of the moment map. This required a vital lemma of Atiyah and Bott [3] which says that this Morse function is, in fact, *perfect*. As a consequence, the cohomology of Hamiltonian G-spaces is completely determined by the critical sets, which in turn are related to the fixed points under the group action. In [25], we prove the following K-theoretic version the Atiyah-Bott lemma:

Lemma (Harada-Landweber)**.** *Let G be a compact connected Lie group, and suppose that $E \to X$ is a* Spin^c *G-vector bundle over a compact G-manifold X. If there exists a subgroup $S^1 \subset G$ acting on E with fixed point set $E^{S^1} = X$ the zero section, then the equivariant Euler class $e_G(E) \in K_G^*(X)$ is not a zero-divisor.*

As an application of this lemma, in [25] we prove the following K-theoretic version of the Kirwan surjectivity theorem:

Theorem (Harada-Landweber: Kirwan Surjectivity)**.** *Let G be a compact connected Lie group. If M is a Hamiltonian G-space with proper moment map $\mu : M \to \mathfrak{g}^*$, and 0 is a regular value of μ, then the K-theory Kirwan map*

$$\kappa : K_G^*(M) \to K_G\big(\mu^{-1}(0)\big) \cong K^*(M /\!/ G)$$

is surjective.

Using Morse theory for components of the moment map $\mu^Z(x) = \langle \mu(x), Z\rangle$ for $Z \in \mathfrak{g}$, we also prove the following results in [25]:

Theorem (Harada-Landweber: Equivariant Formality)**.** *Let G be a compact connected Lie group with $\pi_1(G)$ torsion-free. Then for every compact Hamiltonian G-space M we have a ring isomorphism*

$$K_G^*(M) \otimes_{R(G)} \mathbb{Z} \cong K^*(M).$$

Corollary (Harada-Landweber)**.** *Let G be a compact connected Lie group with $\pi_1(G)$ torsion-free. Then every complex line bundle over a compact Hamiltonian G-space admits a lift of the G-action.*

Having established that the Kirwan map surjects onto the K-theory of the symplectic quotient, our task becomes to compute the kernel of this map. In [26] we address the case of abelian symplectic quotients where $G = T$ is a torus, where we give an explicit construction of the kernel of the Kirwan map, generalizing the rational cohomology techniques of [40].

On the other hand, the nonabelian case is much more difficult. We conjecture the following K-theoretic version of a theorem due to Shaun Martin (see [39] and also [32]), relating the cohomology of the symplectic quotient by a group and by its maximal torus:

Conjecture. *Let G be a compact connected Lie group with $\pi_1(G)$ torsion-free, and let T be a maximal torus in G. If M is a compact Hamiltonian G-space, and 0 is a regular value of both moment maps μ_G and μ_T, then*

$$K^*(M /\!/ G) \cong \frac{K^*(M /\!/ T)^W}{\mathrm{Ann}\big(e_G(G/T)\big)},$$

where W is the Weyl group of G, and the equivariant Euler class $e_G(G/T) \in R(T)$ is the denominator of the Weyl character formula.

So far, we have been able to compute upper and lower bounds on $K^*(M /\!/ G)$, which coincide if we can invert the order of the Weyl group. Our approach to proving this conjecture involves comparing the K-theories of the two symplectic quotients

$$M /\!/ G = \mu_G^{-1}(0)/G \qquad \text{and} \qquad M /\!/ T = \mu_T^{-1}(0)/T$$

via the intermediate space $\mu_G^{-1}(0)/T$. As a first step we must examine the restriction map

$$K_G^*\big(\mu_G^{-1}(0)\big) \longrightarrow K_T^*\big(\mu_G^{-1}(0)\big).$$

This led us to consider in [27] the general case of this restriction map, comparing the G-equivariant and T-equivariant K-theories of a compact G-space. Extending the integral cohomology results of [31], we prove the following:

Theorem. *Let G be a compact connected Lie group with $\pi_1(G)$ torsion-free, and let T be a maximal torus in G. Given a compact G-space X, the G-equivariant K-theory injects via the restriction map to the T-equivariant K-theory as the invariants*

$$K_G^*(X) \cong K_T^*(X)^{\mathcal{I}},$$

with respect to the augmentation ideal $\mathcal{I}$ in the ring $\mathrm{End}_{R(G)} R(T)$, which acts on $K_T^*(X)$ via divided difference operators constructed from the index homomorphisms for all homogeneous differential operators on G/T.

Over a point, this result reduces to the familiar isomorphism $R(G) \cong R(T)^W$ of representation rings. In the general case, the algebra $\mathrm{End}_{R(G)} R(T)$ extends the group ring $\mathbb{Z}(W)$ of the Weyl group. In addition, we have already proved an extension of this result, replacing T with any maximal rank subgroup $H \subset G$, using the results of [23] and my first paper [35]. The maximal rank version will appear in a forthcoming paper.

In the above discussion, we have always required our Lie group to have torsion-free fundamental group. At the level of representation rings, this allows us to avoid projective representations of G with non-trivial cocycles. Without that restriction, these results likely have generalizations in terms of twisted equivariant K-theory, as described in [4, 38].

3.1.2 Long-term plans

In addition to symplectic quotients, I am also interested in analogous quotients in related geometries. I am working with Megumi Harada, Lisa Jeffrey, and Paul Selick (the latter two both at the University of Toronto) to study the K-theory of quasi-Hamiltonian quotients. In the quasi-Hamiltonian case, we have group-valued moment maps in the sense of [1], and quasi-Hamiltonian spaces are finite dimensional analogues of infinite dimensional Hamiltonian loop group spaces. The rational cohomology of Hamiltonian loop group and quasi-Hamiltonian quotients was studied by Bott, Tolman, and Weitsman in [5], and Harada and Selick have studied the K-theory of Hamiltonian loop group quotients in [28]. We believe that the corresponding results for the K-theory of quasi-Hamiltonian spaces will involve twisted versions of K-theory.

In the future, I would like to explore the extensions of my results involving symplectic quotients to other generalized cohomology theories, such as real KO-theory, elliptic cohomology, and cobordism. I have also done some preliminary work with Megumi Harada and Graeme Wilkin (at Johns Hopkins University) to prove the analogous results for the rational cohomology of *hyperkähler* quotients, where there is not one but three symplectic forms, with associated complex structures I, J, K satisfying quaternion relations [30].

3.2 Off-shell supersymmetry in physics

The work described here is a collaborative research project with Charles Doran (University of Alberta, Mathematics), Michael Faux (SUNY Oneonta, Physics), S. James Gates, Jr. (John S. Toll Professor of Physics, University of Maryland), Kevin Iga (Pepperdine University, Mathematics), Tristan Hübsch (Howard University, Physics), and Robert Miller (University of Washington, Mathematics).

In physics, *supersymmetry* refers to theories which are fixed under the super Poincaré group or its variants. The conventional Poincaré group consists of isometries of Minkowski space, encoding Einstein's principle that the laws of physics are the same in all reference frames. In nature, there are two distinct classes of particles, bosons and fermions, which obey essentially the same equations, except for differences in sign. The super Poincaré group is a $\mathbb{Z}_2$-graded extension of the Poincaré group by infinitesimal generators which exchange bosons and fermions, and whose squares are derivatives, the generators of spacetime translations. Supersymmetry is an essential ingredient in string theory, and it has many important applications in mathematics, such as the use of Seiberg-Witten theory in the classification of 4-manifolds.

Mathematically, the simplest representations of the super Poincaré group are the irreducible unitary ones, which can be classified by a supersymmetric Wigner construction (see [19]). These representations are spaces of sections of bundles over covector orbits called *mass shells*, and are called *on-shell* representations. In physics, the simplest representations are *off-shell* representations, where the super Poincaré group acts on spaces of fields, which are sections of bundles over all of spacetime, or more generally objects modeled on sections, such as connections. By imposing equations of motion derived from a Lagrangian, one can constrain an off-shell representation to any mass shell, giving a collection of on-shell representations. The "Fundamental Supersymmetry Challenge" (see [20]) is to reverse this process, extending on-shell representations off-shell.

Currently, the only known examples of off-shell supersymmetry are in spacetime dimensions up to six, and are constructed using superspace, a fermionic extension of spacetime. However, string theory and M-theory require supersymmetry in 10 or 11 dimensions, where supersymmetry is known only on-shell. Finding off-shell extensions of these known on-shell theories would have a significant impact on string theory. While on-shell theories are expressed in terms of fields satisfying dynamical constraints and are difficult to quantize, off-shell theories satisfy supersymmetry without such constraints and are easier to quantize.

The goal of this project is to completely classify all off-shell representations of supersymmetry. Our approach is to consider first representations for one or two spacetime dimensions. Then we study higher dimensional representations by examining their "shadows" in one or two dimensions using a process called *dimensional reduction*. By first classifying the low dimensional representations and then determining under what conditions they "oxidize" to higher dimensions, we hope to obtain our complete classification.

3.2.1 Filtered Clifford supermodules

One of my contributions to this project is to put this physical classification problem on a rigorous mathematical footing. To this end, in [7] I prove that off-shell representations of supersymmetry in one or two spacetime dimensions, with an integral grading corresponding to the physical concept of mass dimension or engineering dimension, correspond to filtered or bifiltered Clifford supermodules:

Theorem (Landweber *et al.*). *There is a one-to-one correspondence between isomorphism classes of:*

- *$\mathbb{Z}$-graded off-shell representations of the super Poincaré algebra $\mathfrak{p}^{1|N}$ and filtered* $\mathrm{Cl}(N)$*-supermodules.*
- *$\mathbb{Z}$-graded off-shell representations of the super Poincaré algebra $\mathfrak{p}^{1,1|p,q}$ and bifiltered* $\mathrm{Cl}(p,q)$*-supermodules.*

This pattern does not generalize to tri-filtrations, etc., due to Lorentz invariance. However, for higher dimensional representations of supersymmetry, we can harness that Lorentz invariance, using it to reconstruct the higher dimensional representation from an equivariant filtered Clifford supermodule. The proof of the following will appear in a forthcoming paper:

Theorem (Landweber *et al.*). *Recall that the odd summand of the super Poincaré algebra $\mathfrak{p}^{n|s}$ is a real spin representation $\mathbb{S}$ of $\mathfrak{spin}(1,n-1)$. Given a filtered* $\mathrm{Cl}(\mathbb{S})$*-supermodule V which is equivariant with respect to the massive little group $\mathfrak{spin}(n-1)$, such that*

1. *the associated graded $\Lambda^*(\mathbb{S})$-supermodule is fully $\mathfrak{spin}(1,n-1)$-equivariant, and*
2. *V admits a $\mathfrak{spin}(1,n-1)$-invariant inner product with respect to which the Clifford action of $\mathbb{S}$ is self-adjoint,*

we can construct a corresponding $\mathbb{Z}$-graded off-shell representation of $\mathfrak{p}^{n|s}$.

In light of these results, the problem of classifying infinite dimensional off-shell representations of supersymmetry becomes the problem of classifying equivariant filtrations of finite dimensional Clifford supermodules. To approach this filtration problem, we plan to consider the moduli space of off-shell representations of supersymmetry, consisting of flags in a Clifford supermodule subject to compatibility conditions, modulo equivalence. From this point of view, we hope to study this moduli space using techniques from algebraic geometry and deformation theory (see [21]), or possibly even the symplectic techniques described above.

3.2.2 Adinkra graphs

In [18], my physics collaborators Faux and Gates introduced combinatorial diagrams called *Adinkras* to encode the data involved in constructing a one dimensional off-shell representation of supersymmetry. An Adinkra is a bipartite, N-regular, N-edge colored graph, with edges labeled by signs and vertices labeled by integral heights satisfying various conditions. Interpreted using my formalism described above, the vertices correspond to a basis for the Clifford supermodule, with the bipartition corresponding to the $\mathbb{Z}_2$-grading. The colored edges give the actions of the Clifford generators, and the heights of the vertices determine the filtration levels. Such diagrams are analogous to root and weight diagrams for Lie groups and their representations. To study Adinkras, we consider the underlying graph separately from the assignment of integral heights to the vertices. For the heights, we prove in [8]:

Theorem. *An Adinkra's height assignment is classified by the subset of local maxima or minima, together with their heights. All height assignments on an Adinkra are related by vertex raising and lowering operations.*

In other words, considering the corresponding digraph with edges directed by ascending heights, the heights are determined by the sources or sinks.

The underlying graph, which we call the *Adinkra topology*, corresponds to an unfiltered representation of the Clifford algebra. In [12] we announce and in [16, 17] we prove rigorously the classification of all possible Adinkra topologies as quotients of hypercube graphs. Furthermore, we discovered a surprising and interesting connection to the theory of error correcting codes. The 2^N vertices of an N-hypercube can be indexed by elements of the vector space $(\mathbb{Z}_2)^N$, i.e., strings of N bits. A linear binary code is a subspace of $(\mathbb{Z}_2)^N$, and such a code is called *doubly-even* if the number of 1's in each codeword is a multiple of 4. We prove the following theorem:

Theorem. *The possible Adinkra topologies are in one-to-one correspondence with the doubly-even linear binary codes.*

We are now classifying and listing all doubly-even codes up to $N = 32$, which is the largest size relevant to supersymmetric physics. We are using the mathematical software Sage for our computations, and the early stages of this program were performed on the 8-processor Mac Pro I purchased with my Bard startup funds, which we ran continuously the entire summer of 2008. The results of these computations are given in [17]. In early 2008, I wrote a portion of an NSF grant proposal, which was funded in late 2008 through the University of Washington. This grant gave our group access to even more powerful computers, and subsequently our doubly-even code classification program was off-loaded to a supercomputer.

In [9], we use Adinkras to explicitly construct the simplest irreducible representations of off-shell supersymmetry out of superfields on superspace, and [15] extends this work, showing how any off-shell Adinkra topology arises using super-differentially constrained superfields. Unconstrained superfields are free supermodules over the odd part of the super Poincaré algebra. This led me to consider the free resolutions of off-shell representations, which can be computed using Lie superalgebra homology and cohomology. I gave this problem to my student Sylvia Naples for her senior project, and she found that the dimensions of the

Lie superalgebra homology are determined by a polynomial, thereby giving a polynomial invariant of Adinkras. She and I are working to turn her senior project into a joint paper.

I also used my work with Clifford algebras and codes as the starting point for two other Bard research projects. I advised Zach Hamaker on his senior project studying constructions of representations of real Clifford algebras, examining the representation theory side of this connection with codes. I also advised Anna Casteen on her MAT academic research project studying variations of doubly-even codes corresponding to indefinite real Clifford algebras.

Using Adinkras, we have found many interesting examples and counterexamples useful in the classification and study of off-shell representations. Our papers [11, 13, 14] describe various physical applications of our Adinkra work. In [10], we give examples of N-extended off-shell representations of one-dimensional supersymmetry, i.e., off-shell representations of $\mathfrak{p}^{1|N}$, with $N = 5$ and $N = 6$ which have the same number of fields at each engineering dimension, but which are not isomorphic. Our Adinkra techniques were particularly helpful here, as the Adinkras for these representations are clearly distinct. Subsequently, in her Bard senior project, Sylvia Naples found a simpler example of this phenomenon with $N = 4$.

We have several papers currently under development. One examines cubical cohomology of Adinkras, showing that the numerical data attached to the Adinkra can be described as:

- The vertex bipartition is a $\mathbb{Z}_2$-valued 0-cochain $p : V \to \mathbb{Z}_2$ satisfying $dp = \mathbf{1}_1$, where $\mathbf{1}_1$ is the $\mathbb{Z}_2$-valued 1-cochain taking the value 1 on each edge.
- The edge signs are a $\mathbb{Z}_2$-valued 1-cochain $s : E \to \mathbb{Z}_2$ satisfying $ds = \mathbf{1}_2$, where $\mathbf{1}_2$ is the $\mathbb{Z}_2$-valued 2-cochain taking the value 1 on each square.
- The vertex heights are a $\mathbb{Z}$-valued 0-cochain $h : V \to \mathbb{Z}$ with $h \equiv p \bmod 2$ and $dh : E \to \{\pm 1\} \subset \mathbb{Z}$.

Using this description, we have found that the edge signs contain information analogous to a spin structure (see [6]). The bipartition could be viewed as being analogous to an orientation, and the vertex heights are then an integral lift of the orientation data.

Another forthcoming paper takes my work described above relating filtered Clifford supermodules to higher dimensional off-shell supersymmetry, and translates it into the language of Adinkras. Although Faux and Gates conceived of Adinkras as a graphical mnemonic for describing supersymmetry reduced to one spacetime dimension, it turns out that we can use Lorentz symmetry to reconstruct the full high dimensional supersymmetry from the Adinkra. We need only two pieces of additional information: First, we put a inner product on the space of fields such that the vertices of the Adinkra correspond to an orthonormal basis. Second, the edge colors must span a real spin representation of the Lorentz group.

In the future, we plan to classify and examine Adinkras in the 10 and 11 spacetime dimensions required for string and M-theories, as well as study the subtleties involved in gauge theores, gauge-invariant theories, and their generalizations. I am also interested in adapting this work to extensions of the super Poincaré algebra, introducing central extensions and considering the superconformal algebra that appears in string theory. In addition, I am curious whether this work can be connected to the results of [23] and Kostant's Euler number multiplets and cubic Dirac operators [34], on which I based my early work [35, 36]. Such connections may prove fruitful for the physical applications of our work, or they may lead to generalizations of our techniques to other Lie superalgebras arising in mathematics.

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4 Service

4.1 Mathematics Program

In my two years at Bard College, I have played an active role in the running of the Mathematics Program. Some of my responsibilities and activities within the program have been:

- Wrote the Mathematics Program budgets for both the 2008–2009 and the 2009–2010 academic years. In addition to supporting expanded student research activities and providing funds for outside speakers, my budget allowed the program to purchase a projector and upgrade the machines in the Albee 100 computer lab so that it may be used as a lab classroom.

- Supervised the Albee 100 computer lab, hiring and coordinating its lab monitors.

- Webmaster for the Mathematics Program web site, revising and updating it with the latest information about the program's courses, activities, and student research, as well as making it more friendly to prospective students.

- Coached the Bard College team for the Putnam Mathematical Competition, an annual exam given in the first week of December to college students throughout the USA and Canada. On the 2008 exam, Bard placed 41st out of 545 institutions, with a record turnout of 24 students, 14 of them scoring at least one point on this extraordinarily difficult exam.

- Involved in the development of the mathematics curriculum, developing applied mathematical courses on Numerical Analysis and Coding Theory, as well as an Advanced Linear Algebra course aimed at students heading to graduate school.

- Member of the Mathematics Program Search Committee which hired John Cullinan, Jim Belk, and Maria Belk. Along with Sam Hsiao, I travelled to San Diego in January 2008 for the annual AMS/MAA Joint Mathematics Meetings to interview approximately twenty job candidates.

- Invited four of my research colleagues to give talks in the Mathematics, Computer Science, and Physics seminar: Michael Faux (SUNY Oneonta), Robert Miller (University of Washington), Charles Doran (University of Alberta), and Megumi Harada (McMaster University). Having such invited speakers exposes our students to topics outside of our standard curriculum and research beyond the college.

- Used my startup funds to purchase a high powered Mac Pro, which I made available to the college community as a server for the mathematical software Sage, as well as two iMacs for the Albee 318 computer lab for Mathematics Program seniors.

- Represented the Mathematics Program in the Academic Programs Fair at the Open House for Accepted Students, April 18, 2009 and April 26, 2008.

In the coming academic year, I am planning to organize a Mathematics Faculty Research Seminar, possibly over lunch one day a week. This will be an informal venue for our faculty to discuss their advanced research, explaining the technical details of their work, in contrast to our existing seminar, where the lectures are aimed at students. With the growth of the Mathematics Program, the research areas of our faculty increasingly overlap, providing opportunities for collaboration which I hope to promote with this new seminar.

4.2 Division of Science, Mathematics, and Computing

Outside of the Mathematics Program, most of my activities in the college have involved the Computer Science and Physics Programs.

- Invited and organized the March 2009 Distinguished Scientist Lecture, given by S. James Gates, Jr., the John S. Toll Professor of Physics and Directory of the Center for String and Particle Theory at the University of Maryland. Shortly after visiting Bard, Gates was appointed to the President's Council of Advisors on Science and Technology. In addition to his DSL, I arranged during his visit a dinner for math and physics faculty and students, a seminar talk followed by a pizza party, a visit to my string theory class, and meetings with students and faculty to discuss research and issues of diversity.

- Taught *Data Structures* in the Computer Science Program Fall 2008, helping cover courses during Bob McGrail's sabbatical. My other interdisciplinary teaching includes my Spring 2008 *Numerical Analysis Lab*, which was crosslisted with Computer Science, and a Fall 2008 Physics tutorial on *Quantum Field Theory* with Marjorie Schillo.

- Member of the Spring 2008 Physics Search Committee which hired Christian Bracher.

- Member of the Fall 2008 Computer Science Search Commitee which hired Keith O'Hara.

- Over the summer of 2009, I worked with Mark Halsey and Matthew Deady to help improve the Physics Program's laboratory facilities, and provide a joint research space with the Mathematics Program. We investigated potential outside grants and other internally funded options.

4.3 The College

- I begin a term on the Library, Bookstore, and Computer Committee in Fall 2009. I have wanted to do this since arriving at Bard, and I am looking forward to using my computing experience to help improve Bard's e-mail system, computer security, and networking infrastructure.

- Member of the Fellowships and Awards Committee. I worked with candidates for the Rhodes, Marshall, Fulbright, and Watson scholarships, as well as reviewing candidates for internal scholarships. I particularly enjoyed meeting with many of Bard's brightest students, helping them revise their application essays and prepare for interviews. In particular, I met several times each with Kit Martin (Watson), Christian Lehmann (Marshall), Abigail Paris (both Marshall and Rhodes), Nicholas Hippensteel (Rhodes), Margaret Aldrich (Marshall), and SongSoo Kim (Watson). I also attended a Marshall Scholarship advisors meeting at Union College on July 10, 2009.

- I will be the invited faculty speaker at the Matriculation Ceremony on August 26, 2009.
- During L&T, I have given lectures on String Theory to the incoming first year students.
- Through BPI, I gave a lecture on String Theory at the Woodbourne Correctional Facility, and I advised Julian Cowell's senior project on quaternions.
- Working with the MAT program, I advised Anna Casteen's Academic Research Project on error correcting codes, and I also gave presentations on LaTeX during the summers of 2008 and 2009.

4.4 The Mathematical Community

Outside of Bard, I remain active in the broader mathematical community, both through my research, and also in reviewing articles and using my computer skills to create tools for mathematicians and scientists.

- I have refereed grant proposals for the National Science Foundation, and articles for the *Journal of Differential Geometry*, *Advances in Mathematics*, *Advances in Theoretical and Mathematical Physics*, and the *Journal of K-Theory*.
- I have written eight signed reviews of articles for *Mathematical Reviews*, available online as MathSciNet (`http://www.ams.org/mathscinet/`).
- Organized a Special Session at the November 2005 AMS meeting in Eugene, OR, and two workshops at the Banff International Research Station in the summer of 2006.
- I host the Symplectic Geometry Conferences wiki on my web site `Cohomology.com`. This is a central location for symplectic geometers to post information about upcoming conferences and workshops, as well as available jobs. I took this over from a conventional web site hosted at the University of Toronto, and by turning it into a wiki, i.e., a collaborative web site, it is much easier for people to add new listings.
- I wrote an iPhone app for browsing and searching the e-print archive at `arXiv.org`. This is a repository of over a half million scholarly articles in Physics, Mathematics, Nonlinear Sciences, Computer Science, Quantitative Biology, Quantitative Finance, and Statistics, which can now be accessed from an iPhone using my app.
- I have written two computer tools for teaching and learning Linear Algebra: the Online Row Reducer and the Linear Algebrator. Both teach the Gauss-Jordan row reduction algorithm, walking the student through it step-by-step. The later is a Mac OS X application that similarly teaches the Gram-Schmidt orthonormalization algorithm.

- I have made minor contributions to the open source Mac OS X application TeXShop, the leading editor for writing mathematical papers using the LaTeX typesetting language. I have also written scripts to aid in importing bibliographical entries into a BibTeX database using the program BibDesk.

- I have started working on a Mac OS X front end for the mathematical software Sage, which I presented at the Sage Days conference at the University of Washington in June 2008. Not yet available to the public, this project is on the back burner until I finish several ongoing research projects.