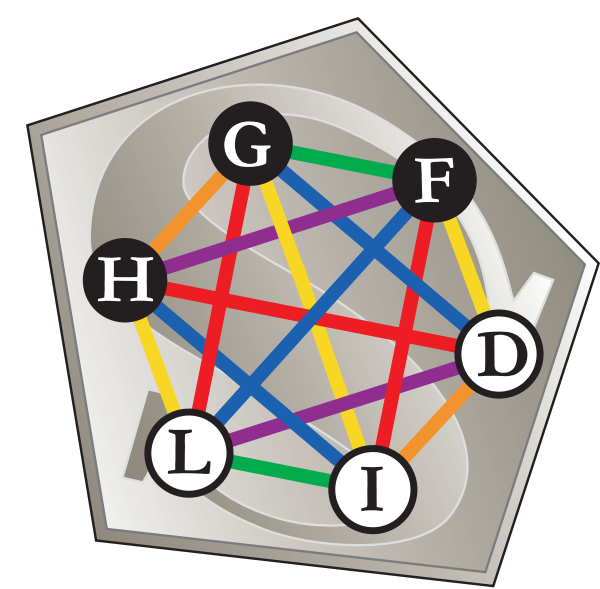


# SUPERSYMMETRY & ADINKRAS

GREGORY D. LANDWEBER  
ASSISTANT PROFESSOR OF MATHEMATICS



In physics, there are two types of elementary particles:

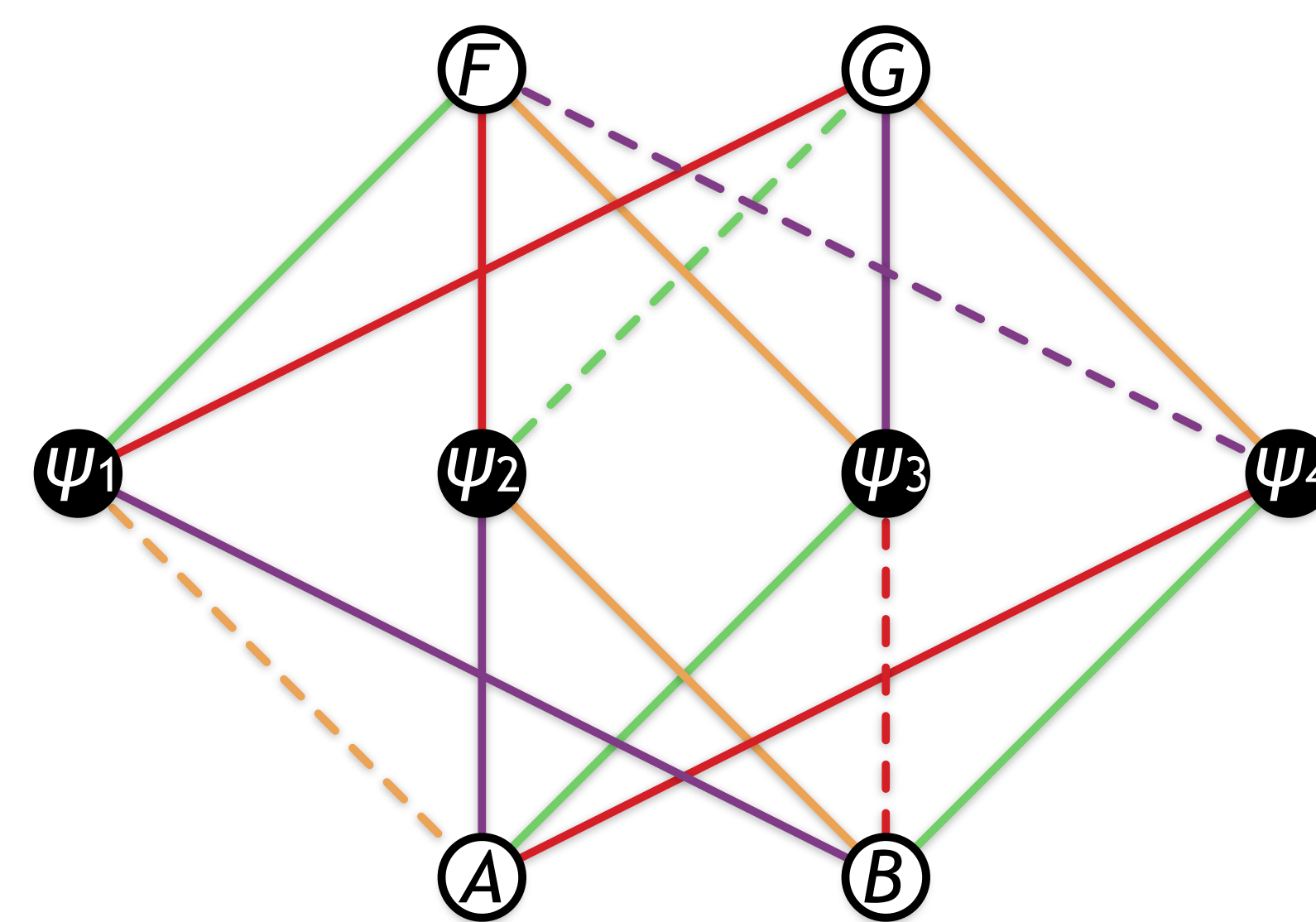
- **Bosons** (such as photons,  $W^\pm$ ,  $Z$ , gluons, gravitons) transmit forces and energy, are often massless, and can pass right through each other.
- **Fermions** (such as electrons, quarks, neutrinos) constitute matter, are usually massive, and obey the Pauli Exclusion Principle.

They behave quite differently, but their descriptions are nearly identical, except for changing a sign from + to -.

**Supersymmetry** is the idea that bosons and fermions are, in fact, interchangeable, and thus any theory of particle physics must contain both types of particles. In practice, physicists propose that the laws of physics are preserved not only under motion in time and space, but also under motion in new supersymmetric directions, exchanging pairs of boson and fermion “superpartners”.

**Adinkras**, introduced by Faux and Gates, are diagrams that describe these intertwined superpartner pairings.

- Vertices correspond to the fields, drawn open for bosons and closed for fermions.
  - Edges correspond to superpartner pairs, colored according to the  $N$  supersymmetric directions.
- So, each edge connects a white vertex to a black vertex, and each vertex has one edge of each color; an Adinkra is a bipartite,  $N$ -regular, edge  $N$ -colored graph. Also, the vertices are assigned integral heights, and the edges are assigned  $\pm 1$  parities (drawn as solid vs. dashed lines), satisfying conditions from the supersymmetry algebra.



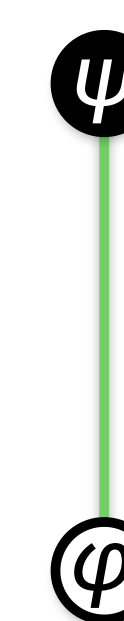
The 4-dimensional chiral Adinkra.

The **supersymmetry algebra** (without central charges) is represented by  $N$  first order linear differential operators  $Q_1, \dots, Q_N$  exchanging bosons and fermions, satisfying the **supersymmetry relations**:

$$Q_i Q_j + Q_j Q_i = 2 \Gamma_{ij},$$

where  $\Gamma$  is a Lorentz-equivariant symmetric matrix of spacetime derivatives.

**Example.** 1-dimensional  $N=1$  supersymmetry. The boson is  $\varphi(t)$ , and the fermion is  $\psi(t)$ . The supersymmetry relation is just  $Q^2 = \partial_t$ , and it is represented by the operator  $Q$ , with  $Q \varphi(t) = \psi(t)$  and  $Q \psi(t) = \partial_t \varphi(t)$ .



Given an Adinkra, the corresponding representation of the supersymmetry algebra is given by the formula:

$$Q_l = \text{up}_l + \sum_j \Gamma_{lj} \text{down}^j,$$

where  $\text{up}_l$  and  $\text{down}^j$  mean following the edge with color  $l$  or  $j$  up or down, with a - sign if the edge is dashed, or 0 if the edge with that color is in the wrong direction.

**Example.** The 4-dimensional chiral Adinkra (at left). In 4-dimensional spacetime, with  $N=4$ , the  $\Gamma$  matrix is:

$\Gamma_{ij}$	1	2	3	4
1	$\partial_t + \partial_z$	0	$\partial_x$	$-\partial_y$
2	0	$\partial_t + \partial_z$	$+\partial_y$	$\partial_x$
3	$\partial_x$	$+\partial_y$	$\partial_t - \partial_z$	0
4	$-\partial_y$	$\partial_x$	0	$\partial_t - \partial_z$

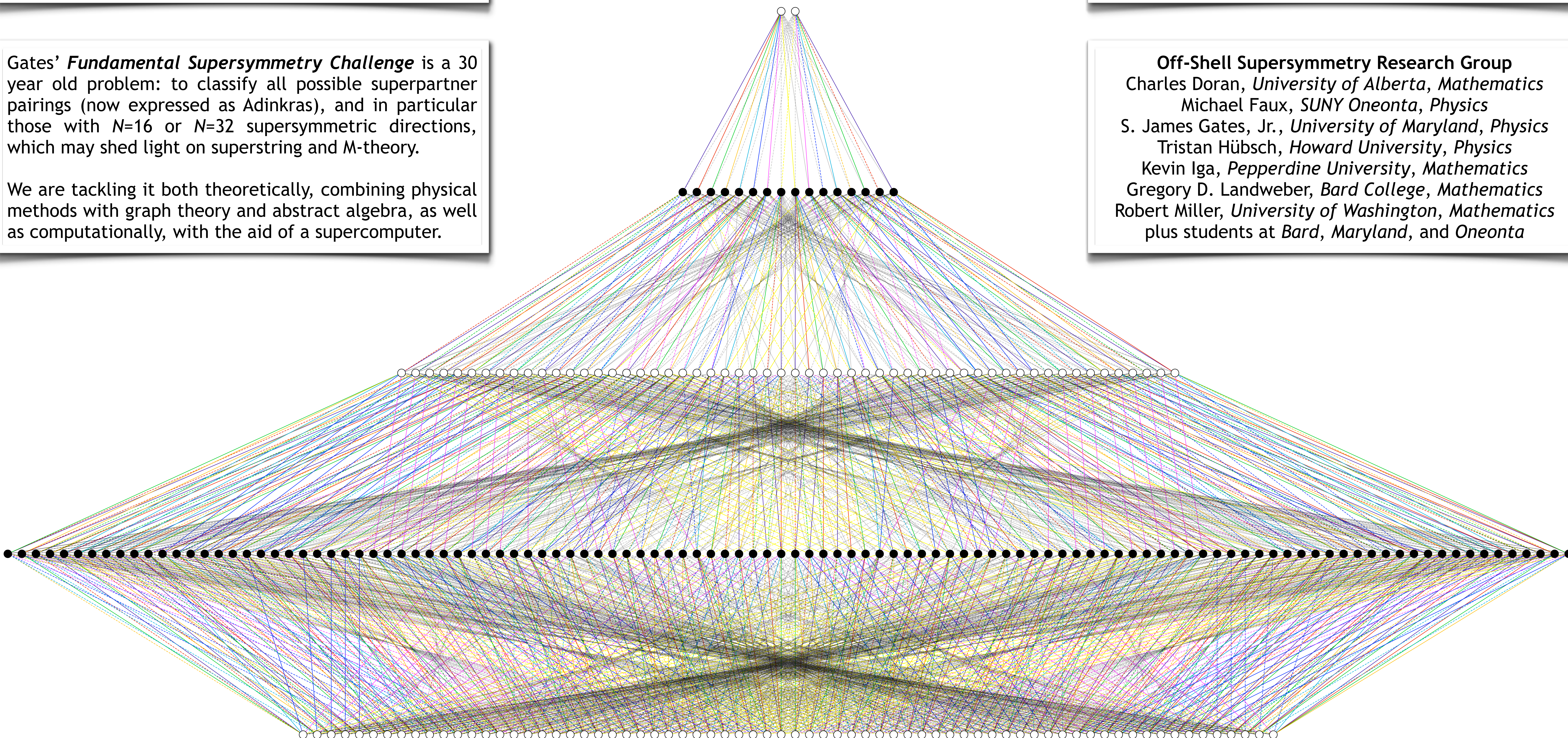
In terms of the linear combination  $\varepsilon^1 Q_1 + \varepsilon^2 Q_2 + \varepsilon^3 Q_3 + \varepsilon^4 Q_4$ , the supersymmetry action on the fields is given by:

$$\begin{aligned} A &\rightarrow -\varepsilon^1 \psi_1 + \varepsilon^2 \psi_2 + \varepsilon^3 \psi_3 + \varepsilon^4 \psi_4, \\ B &\rightarrow +\varepsilon^1 \psi_2 + \varepsilon^2 \psi_1 + \varepsilon^3 \psi_4 - \varepsilon^4 \psi_3, \\ \psi_1 &\rightarrow -\varepsilon^1 (\partial_t + \partial_z) A + \varepsilon^2 (\partial_t + \partial_z) B + \varepsilon^3 (F - \partial_x A + \partial_y B) + \varepsilon^4 (G + \partial_y A + \partial_x B), \\ \psi_2 &\rightarrow \varepsilon^1 (\partial_t + \partial_z) B + \varepsilon^2 (\partial_t + \partial_z) A + \varepsilon^3 (-G + \partial_x B + \partial_y A) + \varepsilon^4 (F - \partial_y B + \partial_x A), \\ \psi_3 &\rightarrow \varepsilon^1 (F + \partial_x A + \partial_y B) + \varepsilon^2 (G + \partial_y A - \partial_x B) + \varepsilon^3 (\partial_t - \partial_z) A - \varepsilon^4 (\partial_t - \partial_z) B, \\ \psi_4 &\rightarrow \varepsilon^1 (G + \partial_x B - \partial_y A) + \varepsilon^2 (-F + \partial_y B + \partial_x A) + \varepsilon^3 (\partial_t - \partial_z) B + \varepsilon^4 (\partial_t - \partial_z) A, \\ F &\rightarrow \varepsilon^1 ((\partial_t + \partial_z) \psi_3 + \partial_x \psi_1 - \partial_y \psi_2) + \varepsilon^2 (-\partial_t - \partial_z) \psi_4 + \partial_y \psi_1 + \partial_x \psi_2 \\ &\quad + \varepsilon^3 (\partial_x \psi_3 - \partial_y \psi_4 + (\partial_t - \partial_z) \psi_1) + \varepsilon^4 (-\partial_y \psi_3 - \partial_x \psi_4 + (\partial_t - \partial_z) \psi_2), \\ G &\rightarrow \varepsilon^1 ((\partial_t + \partial_z) \psi_4 - \partial_x \psi_2 - \partial_y \psi_1) + \varepsilon^2 ((\partial_t + \partial_z) \psi_3 - \partial_y \psi_2 + \partial_x \psi_1) \\ &\quad + \varepsilon^3 (\partial_x \psi_4 + \partial_y \psi_3 - (\partial_t - \partial_z) \psi_2) + \varepsilon^4 (-\partial_y \psi_4 + \partial_x \psi_3 + (\partial_t - \partial_z) \psi_1). \end{aligned}$$

Gates' **Fundamental Supersymmetry Challenge** is a 30 year old problem: to classify all possible superpartner pairings (now expressed as Adinkras), and in particular those with  $N=16$  or  $N=32$  supersymmetric directions, which may shed light on superstring and M-theory.

We are tackling it both theoretically, combining physical methods with graph theory and abstract algebra, as well as computationally, with the aid of a supercomputer.

**Off-Shell Supersymmetry Research Group**  
Charles Doran, *University of Alberta, Mathematics*  
Michael Faux, *SUNY Oneonta, Physics*  
S. James Gates, Jr., *University of Maryland, Physics*  
Tristan Hübsch, *Howard University, Physics*  
Kevin Iga, *Pepperdine University, Mathematics*  
Gregory D. Landweber, *Bard College, Mathematics*  
Robert Miller, *University of Washington, Mathematics*  
plus students at Bard, Maryland, and Oneonta



A computer-generated  $N=16$  Adinkra with 128 bosons and 128 fermions, drawn by the *Adinkramat* by Gregory D. Landweber.