MATH 340: CODING THEORY
TAKE-HOME MIDTERM

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You must solve all of the following problems completely on your own, using only the textbook, your class notes, and your homeworks as references. You must not discuss these problems with the other students, but you may ask questions in class or office hours. Solutions must be turned in by 4 PM on Thursday, October 22. Extensions will be given only in extraordinary circumstances and must be negotiated before the due date if at all possible.

(1) Section 1.8, Exercise 56.

(2) There are precisely two permutation equivalence classes of doubly-even [10,4,4] codes.
  (a) Find them.
  (b) Determine their weight distributions.

  Hint. One of them can be obtained from a code you know quite well by a trivial modification. For the other, think first about even [5,4,2] codes.

(3) Section 1.11, Exercise 67.

(4) I call a code *almost self-orthogonal* if any two of its codewords $\mathbf{x}, \mathbf{y}$ satisfy

$$\mathbf{x} \cdot \mathbf{y} = (\mathbf{x} \cdot \mathbf{x})\,(\mathbf{y} \cdot \mathbf{y}),$$

  and I call a code *almost doubly-even* if the weight of every codeword is congruent to 0 or 1 modulo 4.
  (a) Show that an even almost self-orthogonal code is self-orthogonal.
  (b) Show that an almost doubly-even code which is not even can be obtained from a doubly-even code by adding one row to its generating matrix.
  (c) Show that if an almost self-orthogonal code has a generating matrix with almost doubly-even rows, then the whole code is almost doubly-even.
  (d) Show that almost doubly-even codes are almost self-orthogonal.

  Hint. Adapt the proof of Theorem 1.4.8 to this situation.

(5) Find a parity check matrix for the [16,7,4] code with generating matrix:

$$G = \begin{bmatrix} 1111000000000000 \\ 0011110000000000 \\ 0000111100000000 \\ 0000001111000000 \\ 0000000011110000 \\ 0000000000111100 \\ 0000000000001111 \end{bmatrix}.$$

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