

All problems are equally weighted. Calculators are not allowed. Show all of your work. You may refer to your textbook, your notes, and your homework assignments.

- (1) Consider the differential equation for a damped Harmonic oscillator

$$\frac{d^2x}{dt^2} = -x - \frac{dx}{dt},$$

with initial conditions  $x(0) = 1$ ,  $x'(0) = 0$ . Use two iterations of Euler's method to approximate the value of  $x(1.2)$ .

- (2) Find the general solution to the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= y, \\ \frac{dy}{dt} &= -y + 2x.\end{aligned}$$

- (3) Draw the phase portrait for the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= x + y, \\ \frac{dy}{dt} &= xy.\end{aligned}$$

- (4) Suppose we have found the following two solutions to a second order linear, homogeneous differential equation:

$$\begin{aligned}x(t) &= t^2 - 1, \\ x(t) &= -e^{-t}.\end{aligned}$$

Find the solution to this same differential equation satisfying the initial conditions  $x(0) = 2$ ,  $x'(0) = -1$ .

- (5) The US Mint produces approximately 300 million new quarters each month, replacing old quarters that are cashed in at banks and CoinStar machines. There are around 72 billion total quarters in circulation. For the last 10 years, the US Mint has been issuing state quarters, but that program has just ended, and newly minted quarters revert to the standard design. Assuming that half of the state quarters that were minted are still in circulation right now, how long will it be until only 10% of circulating quarters are state quarters?