Quiz 1 Practice Problems: Permutations
Math 332, Spring 2010

These are not to be handed in. The quiz will be on Tuesday.

1. Write each of the following permutations as a product of disjoint cycles:
   a. \[
   \begin{bmatrix}
   1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
   2 & 3 & 4 & 5 & 1 & 7 & 8 & 6
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
   1 & 3 & 8 & 7 & 6 & 5 & 2 & 4
   \end{bmatrix}
   \]

2. Compute each of the following products:
   a. \((1 2 3 5)(4 1 3)\) b. \((1 3 2 5 6)(2 3)(4 6 5 1 2)\) c. \((1 2)(1 3)(2 3)(1 4 2)\)

3. What is the order of each of the following permutations?
   a. \((1 2 4)(3 5 7)\) b. \((1 2 4)(3 5 6 7)\) c. \((1 2 4)(3 5)\)
   d. \((1 2 4)(3 5 7 8 6 9)\) e. \((1 2 3 4 5 6 7 8 9)(10 11 12 13 14 15)\)

4. What is the order of each of the following permutations?
   a. \[
   \begin{bmatrix}
   1 & 2 & 3 & 4 & 5 & 6 \\
   2 & 1 & 5 & 4 & 6 & 3
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   1 & 2 & 3 & 4 & 5 & 6 & 7 \\
   7 & 6 & 1 & 2 & 3 & 4 & 5
   \end{bmatrix}
   \]

5. Given that \(\alpha = (1 7 5 2 4 8)(3 9)\) and \(\beta = (2 4 9 8 3 7 5)\), compute each of the following:
   a. \(\alpha\beta\) b. \(\alpha^{-1}\) c. \(\beta^{30}\) d. \(\alpha^{40}\)

6. Let \(\beta \in S_7\) and suppose \(\beta^4 = (1 4 3 5 6 7 2)\). Find \(\beta\).

7. Find three elements \(\sigma\) in \(S_9\) with the property that \(\sigma^3 = (1 5 7)(2 8 3)(4 6 9)\).

8. Suppose that \(\beta\) is a 10-cycle. For which integers \(i\) between 2 and 10 is \(\beta^i\) also a 10-cycle?

9. Let \(\alpha = (1 3 5 7 9)(2 4 6)(8 10)\). If \(\alpha^m\) is a 5-cycle, what can you say about \(m\)?

10. Determine whether the following permutations are even or odd.
    a. \((1 3 5)\) b. \((1 3 5 6)\) c. \((1 3 5 6 7)\)
    d. \((1 2)(1 3 4)(1 5 2)\) e. \((1 2 3 4)(3 5 2 1)\)

11. What are the possible orders for the elements of \(S_6\) and \(A_6\)? What about \(S_7\)?

12. Find an element of order 15 in \(A_8\). Does \(A_8\) have an element of order 6?

13. What is the maximum order of any element of \(A_{10}\)?

14. How many elements of order 5 are in \(S_7\)?

15. How many elements of order 4 does \(S_6\) have? What about order 2?

16. In \(S_4\), find a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4. Do the same for \(A_8\).
Answers

1. a. \((1 2 3 4 5)(6 7 8)\) b. \((2 3 8 4 7)(5 6)\)

2. a. \((1 5)(2 3 4)\) b. \((1 2 4)(3 5)\) c. \((1 4 2 3)\)

3. a. 3 b. 12 c. 6 d. 6 e. 18

4. a. 6 b. 12

5. a. \((1 7 2 8 9)(3 5 4)\) b. \((1 8 4 2 5 7)(3 9)\) c. \((2 9 3 5 4 8 7)\) d. \((1 4 5)(2 7 8)\)

6. \(\beta = (1 3 6 2 4 5 7)\)

7. There are actually 18 such elements, all of which are 9-cycles. Three examples include 
\((1 2 4 5 8 6 7 3 9), (1 4 2 5 6 8 7 9 3), \) and \((1 4 8 5 6 3 7 9 2)\).

8. 3, 7, 9

9. We can say that \(m\) is a multiple of 6, but not a multiple of 5. (That is, \(m\) must be congruent to 6, 12, 18, or 24 modulo 30.)

10. a. even b. odd c. even d. odd e. even

11. For \(S_6\), the possible orders are 1, 2, 3, 4, 5, 6; for \(A_6\), 1, 2, 3, 4, 5; for \(S_7\), 1, 2, 3, 4, 5, 6, 7, 10, 12.

12. \((1 2 3 4 5)(6 7 8)\) has order 15, and \((1 2)(3 4)(5 6 7)\) has order 6.

13. \((1 2 3 4 5 6 7)(8 9 10)\) has order 21.

14. 504

15. 180; 75

16. In \(S_4\), \(\langle (1 2 3 4) \rangle\) is a cyclic subgroup of order 4, and \(\{e, (1 2), (3 4), (1 2)(3 4)\}\) is a Klein 4-group. In \(A_8\), \(\langle (1 2 3 4)(5 6 7 8) \rangle\) is a cyclic subgroup of order 4, and \(\{e, (1 2)(3 4), (5 6)(7 8), (1 2)(3 4)(5 6)(7 8)\}\) is a Klein 4-group. (Many other answers are possible.)